Name:______________________

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Support Sheet (*continued*)

1. A school organizes an outdoor trip to Gaspésie. A group of 54 students at most can participate in this activity. Organizers plan to rent at least 10 tents. Two types of tents are recommended: large tents that cost \$50 and can accommodate six people, and small tents that cost \$30 and can accommodate four people. There will be at least one large tent used to store the luggage. How many large and small tents will the organizers have to rent if the organizers want to minimize rental costs?

a) Define the variables (x and y) in this situation.

x: the number of large tents

y: the number of small tents

b) State the rule representing the function Z to be optimized as a function of x and y.

Z = 50x + 30y

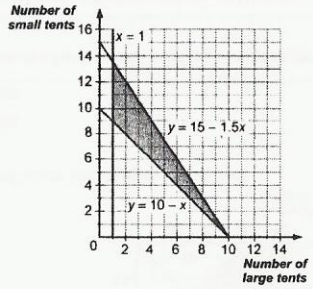
c) Translate algebraically the constraints of this situation.

$x + y \geq 10$

$6(x - 1) + 4y \leq 54$ or $6x + 4y \leq 60$

$x \geq 1$

$y \geq 0$



d) Draw the polygon of constraints of the system of linear inequalities that you found in **c)** in the Cartesian plane above.

e) Specify the vertices of the polygon of constraints.

A(10, 0), B(1, 9) and C(1, 13.5).

f) Determine the number of small tents and large tents that will have to be rented in order to minimize rental costs.

The scanning line technique or the vertices of a polygon of constraints technique can be used to optimize the function Z, since it can be represented by a line $\left(y = -\frac{5}{3}x + \frac{Z}{30}\right)$.

Vertex	Rental costs (Z = 50x + 30y)
A(10, 0)	Z = 50(10) + 30(0) = 500 + 0 = \$500
B(1, 9)	Z = 50(1) + 30(9) = 50 + 270 = \$320
C(1, 13.5)	Z = 50(1) + 30(13.5) = 50 + 405 = \$455

To minimize their rental costs, organizers have to rent one large tent and nine small tents.

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Support Sheet (*continued*)

2. A city neighbourhood is offering bicycle repair courses that are limited to a maximum of 12 participants. There are two types of courses offered: a one-hour beginner's course that costs \$16 and a two-and-a-half hour advanced course that costs \$12. The municipality lends the required equipment for a maximum of 18 hours per week. It was determined that that the number of people registered for the advanced course had to be at most equal to twice the number registered for the beginner's course. If the municipality wants to maximize its revenue, how many people will it allow to register for each course?

a) Define the variables (x and y) in this situation.

x: the number of people registered for the beginner's course

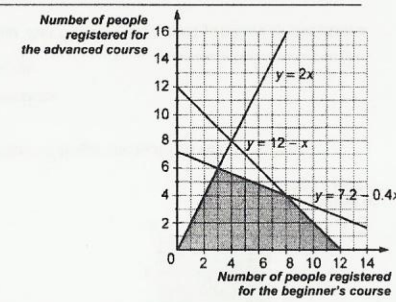
y: the number of people registered for the advanced course

b) State the rule representing the function Z to be optimized as a function of x and y.

Z = 16x + 12y

c) Translate algebraically the constraints of this situation.

$x + y \leq 12$	$x \geq 0$
$y \leq 2x$	$y \geq 0$
$x + 2.5y \leq 18$	



d) Draw the polygon of constraints of the system of linear inequalities that you found in **c)** in the Cartesian plane above.

e) Specify the vertices of the polygon of constraints.

A(0, 0), B(3, 6), C(8, 4) and D(12, 0).

f) Determine the number of people registered for each course that will allow the municipality to maximize its revenue.

The scanning line technique or the vertices of a polygon of constraints technique can be used to optimize the function Z, since it can be represented by a line $\left(y = -\frac{4}{3}x + \frac{Z}{12}\right)$.

Vertex	Revenue (Z = 16x + 12y)
A(0, 0)	Z = 16(0) + 12(0) = 0 + 0 = \$0
B(3, 6)	Z = 16(3) + 12(6) = 48 + 72 = \$120
C(8, 4)	Z = 16(8) + 12(4) = 128 + 48 = \$176
D(12, 0)	Z = 16(12) + 12(0) = 192 + 0 = \$192

To maximize its revenue, the municipality will have to register a minimum of 12 people for the beginner's course and does not need to register anyone for the advanced course.

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Additional Exercises

1. The Little Oven Bakery offers its customers two types of pastries: cookies and tarts. It always sells at least as many cookies as tarts. In a given week, it never bakes fewer than 60 or more than 90 pastries. The number of cookies combined with twice the number of tarts sold does not exceed 120 pastries in all. If the cookies and tarts sell for $3 and $5 respectively, and the production costs are $35, how many cookies and tarts must the bakery produce to maximize its profit?

a) Define the variables (x and y) in this situation.

x: the number of cookies

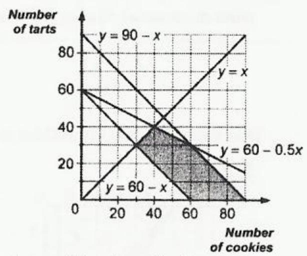
y: the number of tarts

b) State the rule representing the function to be optimized.

$Z = 3x + 5y - 35$

c) Translate algebraically the constraints of this situation.

$x \geq y$	$x + 2y \leq 120$
$x + y \leq 90$	$x \geq 0$
$x + y \geq 60$	$y \geq 0$



d) Draw the polygon of constraints of the system of linear inequalities found in **c)** in the Cartesian plane above.

e) Specify the vertices of the polygon of constraints.

A(90, 0), B(60, 0), C(30, 30), D(40, 40) and E(60, 30).

f) Determine the number of each type of pastry that the bakery must produce to maximize its profit.

The scanning line technique or the vertices of a polygon of constraints technique can be used to optimize the function Z, since it can be represented by a line $\left(y = -\frac{3}{5}x + \frac{Z}{5} + 7\right)$.

Vertex	Profit ($Z = 3x + 5y - 35$)
A(90, 0)	$Z = 3(90) + 5(0) - 35 = 270 + 0 - 35 = \$235$
B(60, 0)	$Z = 3(60) + 5(0) - 35 = 180 + 0 - 35 = \$145$
C(30, 30)	$Z = 3(30) + 5(30) - 35 = 90 + 150 - 35 = \$205$
D(40, 40)	$Z = 3(40) + 5(40) - 35 = 120 + 200 - 35 = \$285$
E(60, 30)	$Z = 3(60) + 5(30) - 35 = 180 + 150 - 35 = \$295$

To maximize its profit, the bakery must produce 60 cookies and 30 tarts per week.

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Additional Exercises (*continued*)

2. A company manufactures guitars. It can produce at most 24 electric guitars and 30 acoustic guitars per month. An electric guitar sells for $80 and an acoustic guitar sells for $100. Given the sales figures, it is specified that triple the number of electric guitars combined with quadruple the number of acoustic guitars must not exceed 144 per month. If the company's objective is to maximize its revenue, how many of each type of guitar should it produce?

a) Define the variables (x and y) in this situation.

x: the number of electric guitars

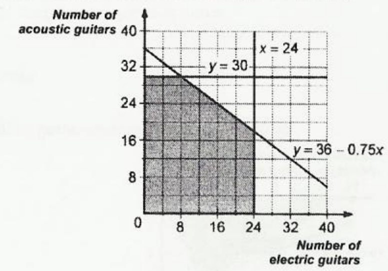
y: the number of acoustic guitars

b) State the rule representing the function to be optimized.

$Z = 80x + 100y$

c) Translate algebraically the constraints of this situation.

$x \leq 24$	$x \geq 0$
$y \leq 30$	$y \geq 0$
$3x + 4y \leq 144$	



d) Draw the polygon of constraints of the system of linear inequalities that you found in **c)** in the Cartesian plane above.

e) Specify the vertices of the polygon of constraints.

A(0, 0), B(0, 30), C(8, 30), D(24, 18) and E(24, 0).

f) Determine the number of each type of guitar that the company must produce to maximize its revenue.

The scanning line technique or the vertices of a polygon of constraints technique can be used to optimize the function Z, since it can be represented by a line $\left(y = -\frac{4}{5}x + \frac{Z}{100}\right)$.

Vertex	Revenue ($Z = 80x + 100y$)
A(0, 0)	$Z = 80(0) + 100(0) = 0 + 0 = \$0$
B(0, 30)	$Z = 80(0) + 100(30) = 0 + 3000 = \$3,000$
C(8, 30)	$Z = 80(8) + 100(30) = 640 + 3000 = \$3,640$
D(24, 18)	$Z = 80(24) + 100(18) = 1920 + 1800 = \$3,720$
E(24, 0)	$Z = 80(24) + 100(0) = 1920 + 0 = \$1,920$

To maximize its revenue, the company will have to produce 24 electric guitars and 18 acoustic guitars per month.

Name:______________________________

Date:__________________ Group:________

Additional Exercises (*continued*)

3. The organizing committee of a neighbourhood party must rent tables for the guests and wants at most 8 tables. A company rents two types of tables: those that seat 20 people and those that seat 50 people. The first type costs \$20 per table and the second type costs \$40 per table. A total rental budget of \$280 was set aside. Members of the organizing committee want to ensure the greatest possible number of seats without going over budget.

a) Define the variables (x and y) in this situation.

x: the number of tables that seat 50 people

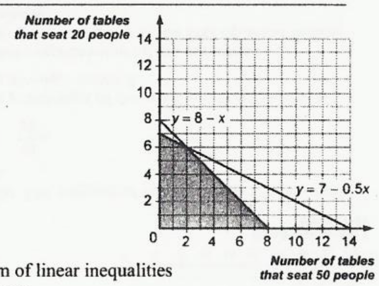
y: the number of tables that seat 20 people

b) State the rule representing the function to be optimized.

Z = 50x + 20y

c) Translate algebraically the constraints of this situation.

$x + y \leq 8$	$x \geq 0$
$20x + 40y \leq 280$	$y \geq 0$



d) Draw the polygon of constraints of the system of linear inequalities that you found in **c)** in the Cartesian plane above.

e) Specify the vertices of the polygon of constraints.

A(0, 0), B(0, 7), C(2, 6) and D(8, 0).

f) Determine the number of each type of table that would seat the maximum number of guests.

The scanning line technique or the vertices of a polygon of constraints technique can be used to optimize the function Z, since it can be represented by a line $\left(y = -\frac{5}{2}x + \frac{Z}{20}\right)$.

Vertex	Number of guests (Z = 50x + 20y)
A(0, 0)	Z = 50(0) + 20(0) = 0 + 0 = 0 guests
B(0, 7)	Z = 50(0) + 20(7) = 0 + 140 = 140 guests
C(2, 6)	Z = 50(2) + 20(6) = 100 + 120 = 220 guests
D(8, 0)	Z = 50(8) + 20(0) = 400 + 0 = 400 guests

To maximize the number of guests, the organizing committee must rent eight tables that seat 50 people and no tables that seat 20 people.

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Additional Exercises (*continued*)

4. François and Danny want to use markers to make a giant poster featuring the logo of their favourite sports team, the Montréal Canadiens. To reproduce the logo, they need at most five times more red than blue. The area to colour will be at most 240 cm^2. The two friends estimate that a blue marker, which costs \$1, will cover 20 cm^2 and that a red marker, which costs \$2, will cover 30 cm^2. They already bought two of each colour marker. In total, how many of each colour marker will they have to buy if they want to minimize the cost of their project?

a) Define the variables (x and y) in this situation.

x: the total number of blue markers bought

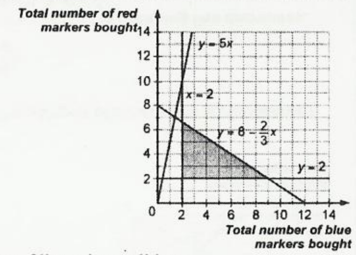
y: the total number of red markers bought

b) State the rule representing the function to be optimized.

Z = x + 2y

c) Translate algebraically the constraints of this situation.

$y \leq 5x$	$x \geq 2$
$20x + 30y \leq 240$	$y \geq 2$



d) Draw the polygon of constraints of the system of linear inequalities that you found in **c)** in the Cartesian plane above.

e) Specify the vertices of the polygon of constraints.

A(2, 2), $B\left(2, \frac{20}{3}\right)$ ***and C(9, 2).***

f) Determine the number of each colour marker that the two friends must buy to minimize the cost of creating this poster.

The scanning line technique or the vertices of a polygon of constraints technique can be used to optimize the function Z, since it can be represented by a line $\left(y = -\frac{1}{2}x + \frac{Z}{2}\right)$.

Vertex	Cost (Z = x + 2y)
A(2, 2)	Z = 2 + 2(2) = 2 + 4 = \$6
$B\left(2, \frac{20}{3}\right)$	$Z = 2 + 2\left(\frac{20}{3}\right) = 2 + \left(\frac{40}{3}\right) = \$15.33$
C(9, 2)	Z = 9 + 2(2) = 9 + 4 = \$13

To minimize the cost of creating the poster, the two friends must use only the four markers that they already bought.

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Knowledge Evaluation Sheet 1

1. A small workshop produces two very popular skateboard models. Model 1 requires 20 minutes of sawing and 40 minutes of sanding, while Model 2 requires 25 minutes of sawing and 35 minutes of sanding. Model 1 sells for \$55 and model 2 sells for \$60. The workshop is open five days a week. The saw cannot be used more than 2 h 40 min per day, while the sander cannot be used for more than 4 h per day. The workshop owner sells at least 20 Model 2 skateboards per week. The weekly number of Model 2 skateboards sold is greater than twice the number of Model 1 skateboards sold. The workshop owner claims that the maximum weekly revenue he can generate is \$2,000. Is he right? Justify your answer.

No, the owner is not right, since the maximum weekly revenue is \$1,950.

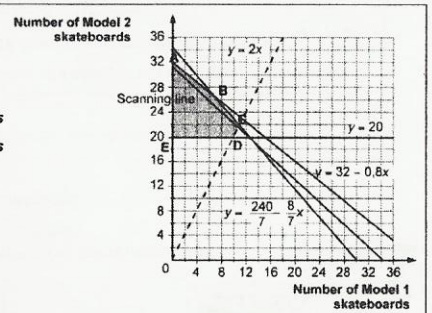
Procedure using the scanning line technique:

x: the number of Model 1 skateboards

y: the number of Model 2 skateboards

System of inequalities:

1) $20x + 25y \le 800$
2) $40x + 35y \le 1200$
3) $y > 2x$
4) $y \ge 20$
5) $x \ge 0$



The function to be optimized is $Z = 55x + 60y$. The polygon of constraints is represented by the polygon ABCDE on the graph above.

The equation of the scanning line is $y = \frac{Z}{60} - \frac{11}{12}x$.

The value of Z increases as the value of the y-intercept of the scanning line increases. The maximum value is found when passing through vertex B.

Vertex B is the point of intersection of the lines represented by the equations $20x + 25y = 800$ and $40x + 35y = 1200$. Isolating the variable y in the first equation results in $y = 32 - 0.8x$. Through substitution, you obtain:

$40x + 35(32 - 0.8x) = 1200$

$12x = 80$

$x = \frac{20}{3}$ or $x \approx 6.67$

$y = 32 - 0.8\left(\frac{20}{3}\right)$

$y = \frac{80}{3}$ or $y \approx 26.67$

The coordinates of vertex B are (6.67, 26.67). Integral values must be used. The possible points belonging to the polygon of constraints are (6, 26), (6, 27) and (7, 26). The workshop must produce 6 Model 1 skateboards and 27 Model 2 skateboards to maximize its revenue. The maximum revenue will be \$1,950, that is, 55 x 6 + 60 x 27.

Name:________________________________

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Knowledge Evaluation Sheet 1 *(continued)*

2. An apple grower wants to plant two varieties of apple trees on a newly purchased property. She claims that variety **A** will produce an average of 320 apples per year, while variety **B** will produce an average of 400 apples per year. Variety **A** sells for \$35 and variety **B** sells for \$48 per tree. Since the two varieties of apples will not be harvested at the same time of year, the apple grower wants to have similar numbers of the two varieties of trees. The number of each variety of tree cannot correspond to more than two-thirds the total number of trees. Determine the number of each variety of tree that the apple grower must purchase if she wants to minimize the overall purchase price, knowing that the projected overall annual production is at least 10 000 apples and that there must be fewer than 45 trees on the property.

To minimize the overall purchase price, the apple grower must buy 19 variety A apple trees and 10 variety B apple trees.

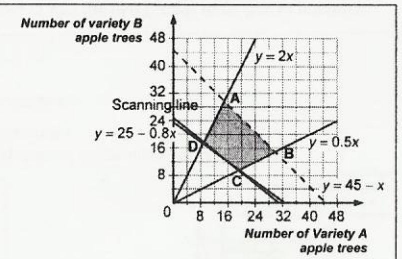
Procedure using the scanning line technique:

x: the number of variety A apple trees

y: the number of variety B apple trees

System of inequalities:

1) $320x + 400y \ge 10\,000$
2) $x + y < 45$
3) $x \le 2y$
4) $y \le 2x$
5) $x \ge 0$
6) $y \ge 0$



The function to be optimized is $Z = 35x + 48y$. The polygon of constraints is represented by the quadrilateral ABCD on the graph above.

The equation of the scanning line is $y = \frac{Z}{48} - \frac{35}{48}x$.

The value of Z decreases as the value of the y-intercept of the scanning line decreases. The minimum value is found when passing through vertex C.

Vertex C is the point of intersection of the lines represented by the equations $320x + 400y = 10\,000$ and $x = 2y$. Substituting x with 2y in the first equation results in:

$320 \cdot 2y + 400y = 10\,000$

$1040y = 10\,000$

$y = \frac{125}{13}$ or $y \approx 9.62$

$x = 2 \cdot \frac{125}{13}$

$x = \frac{250}{13}$ or $x \approx 19.23$

The coordinates of vertex C are (19.23, 9.62). Integral values must be used. The possible points belonging to the polygon of constraints are (19, 10) and (20, 10). The apple grower must buy 19 variety A apple trees and 10 variety B apple trees to minimize the purchase price. The minimum purchase price is therefore \$1,145, that is, 35 x 19 + 48 x 10.

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Knowledge Evaluation **Sheet 2**

1. Zoe owns a small café where she sells sandwiches, including two types with chicken: a chicken salad sandwich and a club sandwich. The chicken salad sandwich contains 110 g of chicken while the club sandwich contains 140 g of chicken. Zoe always sells more than 25 chicken sandwiches per day and the number of club sandwiches sold is always equal to or greater than the number of chicken salad sandwiches sold. A chicken salad sandwich sells for \$4.50 and a club sandwich sells for \$5.25. Zoe claims that the maximum daily income that she can generate by selling these two types of sandwiches, knowing that she has 3.5 kg of chicken available per day, is approximately \$135. Is she right? Justify your answer.

Yes, Zoe is right, since her maximum income is \$136.50 per day.

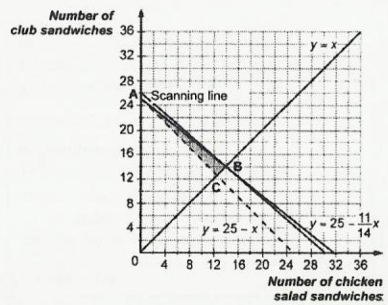
Procedure using the scanning line technique:

x: the number of chicken salad sandwiches

y: the number of club sandwiches

System of inequalities:

1) $110x + 140y \leq 3500$
2) $x + y > 25$
3) $y \geq x$
4) $x \geq 0$
5) $y \geq 0$



The function to be optimized is $Z = 4.5x + 5.25y$. The polygon of constraints is represented by the triangle ABC on the graph above.

The equation of the scanning line is $y = \frac{4}{21}Z - \frac{6}{7}x$.

The value of Z increases as the value of the y-intercept of the scanning line increases. The maximum value is found when passing through vertex B.

Vertex B is the point of intersection of the lines represented by the equations $110x + 140y = 3500$ and $y = x$. Substituting x with y in the first equation results in:

$$110y + 140y = 3500$$
$$250y = 3500$$
$$y = 14$$
$$x = 14$$

The coordinates of vertex B are (14, 14). Zoe must prepare 14 of each type of sandwich if she wants to maximize her income. Her maximum income is \$136.50, that is, 4.5 x 14 + 5.25 x 14.

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Knowledge Evaluation **Sheet 2 (*continued*)**

2. Martin is a pastry chef and makes two types of cakes for Valentine's Day: a round cake and a rectangular cake. The ingredients for the round cake cost \$7.50 and the ingredients for the rectangular cake cost \$8. The round cake requires 300 g of icing and the rectangular cake requires 440 g of icing. The round cake sells for \$13 and the rectangular cake sells for \$16. Determine the number of each type of cake that Martin must make if he wants to generate the greatest total return from sales in one day. The cost of ingredients must be at most \$360 and Martin has 16 kg of icing available. Moreover, he notices that he always sells at least 20% more round cakes than rectangular cakes.

Martin must make 34 round cakes and 13 rectangular cakes.

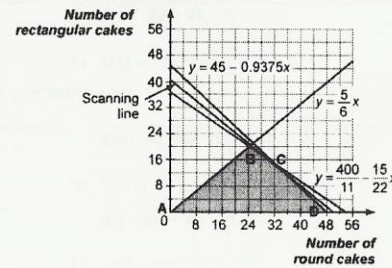
Procedure using the scanning line technique:

x: the number of round cakes

y: the number of rectangular cakes

System of inequalities:

1) $7.5x + 8y \leq 360$
2) $300x + 440y \leq 16\ 000$
3) $x \geq 1.2y$
4) $x \geq 0$
5) $y \geq 0$



The function to be optimized is $Z = 13x + 16y$. The polygon of constraints is represented by the quadrilateral ABCD on the graph above.

The equation of the scanning line is $y = \frac{Z}{16} - \frac{13}{16}x$.

The value of Z increases as the value of the y-intercept of the scanning line increases. The maximum value is found when passing through vertex C.

Vertex C is the point of intersection of the lines represented by the equations $7.5x + 8y = 360$ and $300x + 440y = 16\ 000$. Isolating the variable y in the first equation results in $y = 45 - 0.9375x$. Through substitution, you obtain:

$$300x + 440(45 - 0.9375x) = 16\ 000$$
$$3800 = 112.5x$$
$$x = \frac{304}{9} \text{ or } x \approx 33.78$$

$$y = 45 - \frac{15 \cdot \frac{304}{9}}{16}$$
$$y = \frac{40}{3} \text{ or } y \approx 13.33$$

The coordinates of vertex C are (33.78, 13.33). Integral values must be used. The possible points belonging to the polygon of constraints are (33, 13) and (34, 13). Martin must bake 34 round cakes and 13 rectangular cakes if he wants to generate the greatest total return from sales in one day. The maximum revenue will therefore be \$650, that is, 13 x 34 + 16 x 13.