

General Form of a Circle.

Expanding the standard form of a circle, gives us the general form.

Recall: Remarkable identities (sec 4)/ Completing square (sec 4)

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\Leftrightarrow \boxed{x^2 + y^2 + ax + by + c = 0}$$

Ex:

$$(x+3)^2 + (y-2)^2 = 25 \quad [\text{Standard}]$$

$$x^2 + 6x + 9 + y^2 - 4y + 4 = 25$$

Recall:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$x^2 + y^2 + 6x - 4y + 9 + 4 - 25 = 0$$

$$\boxed{x^2 + y^2 + 6x - 4y - 12 = 0} \quad \text{General}$$

FROM GENERAL TO STANDARD.

① Group the x and y terms together.

$$x^2 + 6x + y^2 - 4y - 12 = 0$$

Recall: A trinomial perfect square is completed by adding $\left(\frac{b}{2a}\right)^2$

$$ax^2 + bx + c = 0 \quad \text{General Quadratic}$$

$$x^2 + 6x + 9 + y^2 - 4y + 4 - 12 - 9 - 4 = 0$$

Now Re-write your x-terms and y -terms as a perfect square trinomial. Simplify the numbers, moving them to the rights side of the equation. This gives you the equation of a circle in STANDARD FORM.

$$x^2 + 6x + 9 + y^2 - 4y + 4 - 12 - 9 - 4 = 0$$

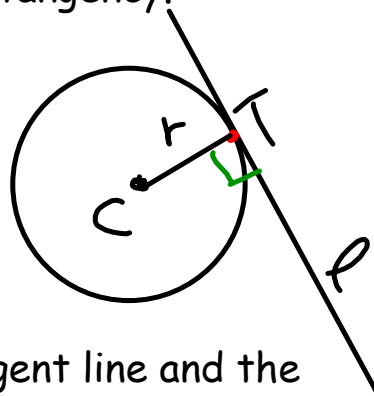
$$(x+3)^2 + (y-2)^2 - 25 = 0$$

$$\boxed{(x+3)^2 + (y-2)^2 = 25} \quad \text{Standard Form.}$$

Line Tangent to a Circle

A line is tangent to a circle, if it intersects the circle in only one point. This is called the point of tangency.

- The angle between the radius and the tangent line is always a right angle.



- The distance between the tangent line and the center of the circle is equal to the radius of the circle.

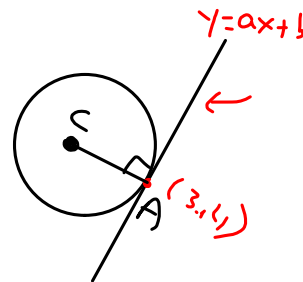
Ex: P 329 of WB

Q18: Find the equation of the tangent to the circle with equation

$$x^2 + y^2 - 2x + 4y - 35 = 0$$

at the point A (3,4)?

- 1) Find eq: Standard form.
- 2) Find coordinates of C.
- 3) Find slope from C to A
- 4) Find equation of the tangent line.



$$x^2 + y^2 - 2x + 4y - 35 = 0$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 - 35 - 1 - 4 = 0$$

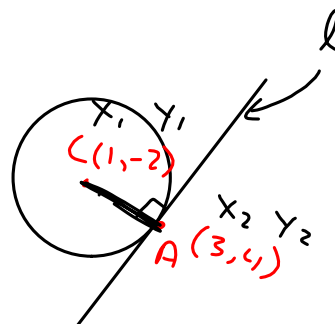
$$(x-1)^2 + (y+2)^2 = 40$$

$$C(1, -2)$$

Slope $\frac{y_2 - y_1}{x_2 - x_1}$

$$CA = \frac{4 - (-2)}{3 - 1}$$

$$= \frac{6}{2} = 3$$



$$\text{slope of } l = -\frac{1}{3}$$

$$y = ax + b$$

$$(3, 4) \Rightarrow y = -\frac{1}{3}x + b$$

$$4 = -\frac{1}{3}(3) + b$$

$$4 = -1 + b$$

$$4 + 1 = b$$

$$b = 5$$

$$\text{Equation of } l : y = -\frac{1}{3}x + 5$$

Work

P326, 327

Q10, 11, 12, 13, 14

P328, 329

Q16, 19, 20, 21