Hyperbola

Definition: A hyperbola is a curve where the absolute value of the difference of the distance between any point on the curve and two fixed points, called foci, is constant.

Equation:

The equation of a hyperbola is in the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1$$

- The coordinates of the center are (0,0)
- The lines associated with the equations: $y = \frac{b}{a}x$, $y = \frac{-b}{a}x$ are the asymptotes of the curve
- The relationship among the values of the parameters a,b and the distance c (between the center of hyperbola and one of its foci) is represented by

$$c^2 = a^2 + b^2$$

if the equation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (case 1)

- The coordinates of the vertices are (a,0) and (-a,0)
- The foci are located on the x-axis and their coordinates are (c,0) and (-c,0)

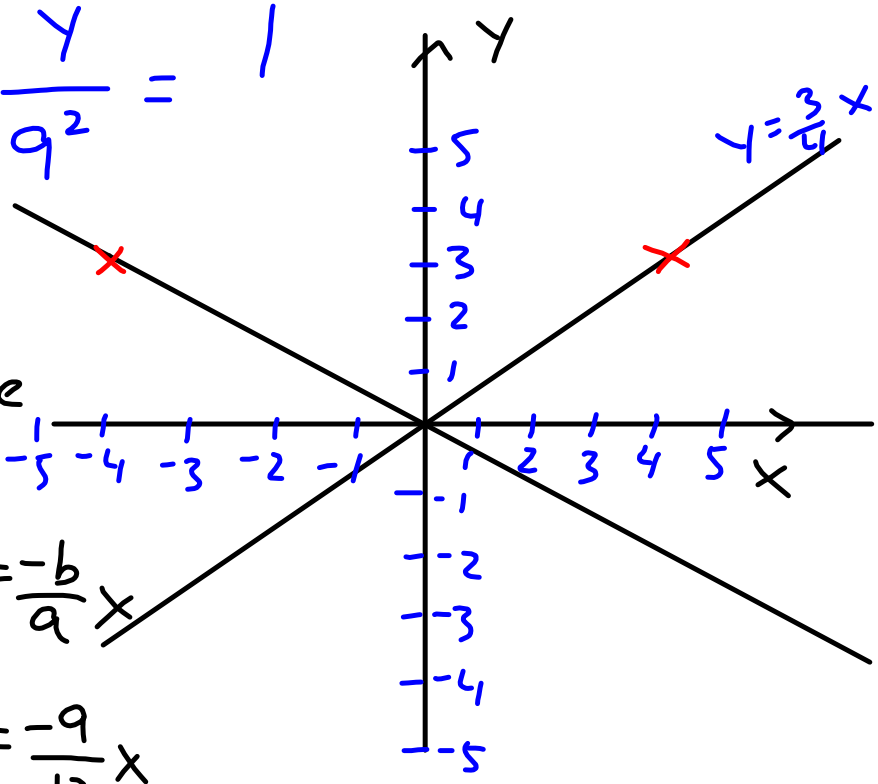
if the equation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ (case 2)

- The coordinates of the vertices are (0,b) and (0,-b)
- The foci are located on the y-axis, and their coordinates are (0,c) and (0,-c)

Case 1:

$$\frac{x^2}{12^2} - \frac{y^2}{9^2} = 1$$

sketch:



① sketch the asymptotes

$y = \frac{b}{a}x$ and $y = \frac{-b}{a}x$

$y = \frac{9}{12}x \quad y = \frac{-9}{12}x$

$y = \frac{3}{4}x \; , \; y = \frac{-3}{4}x$

Coordinates of the vertices.

(12,0) and (-12,0)

Coordinate of the foci

(15,0) (-15,0)

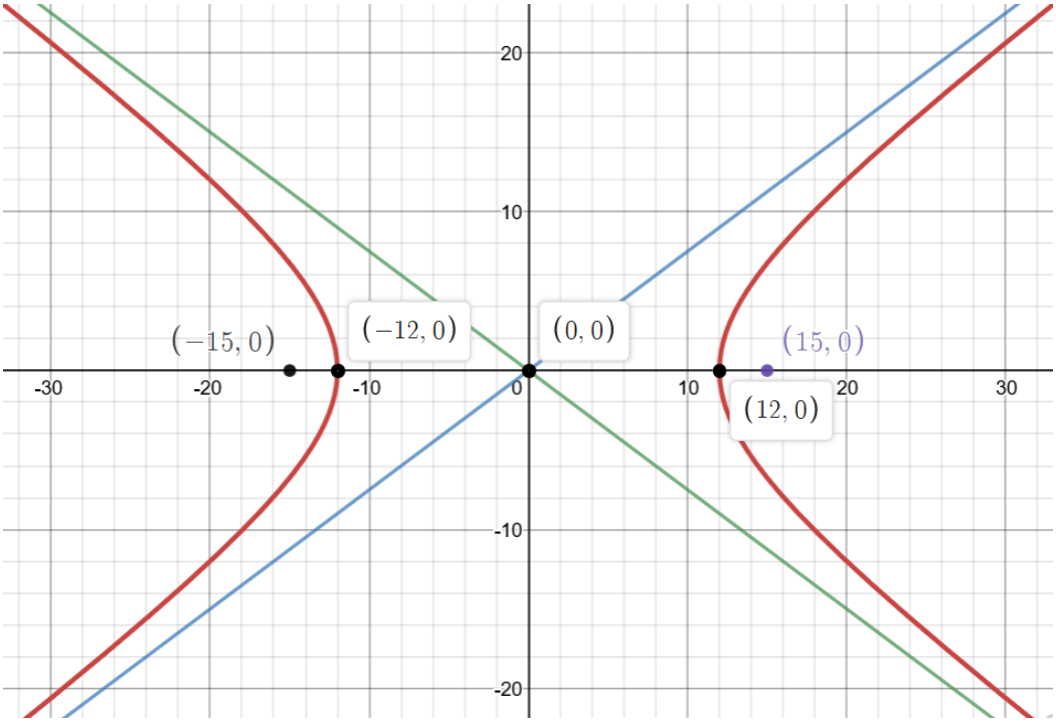
$c^2 = a^2 + b^2$

$c^2 = 12^2 + 9^2$

$c = \sqrt{225}$

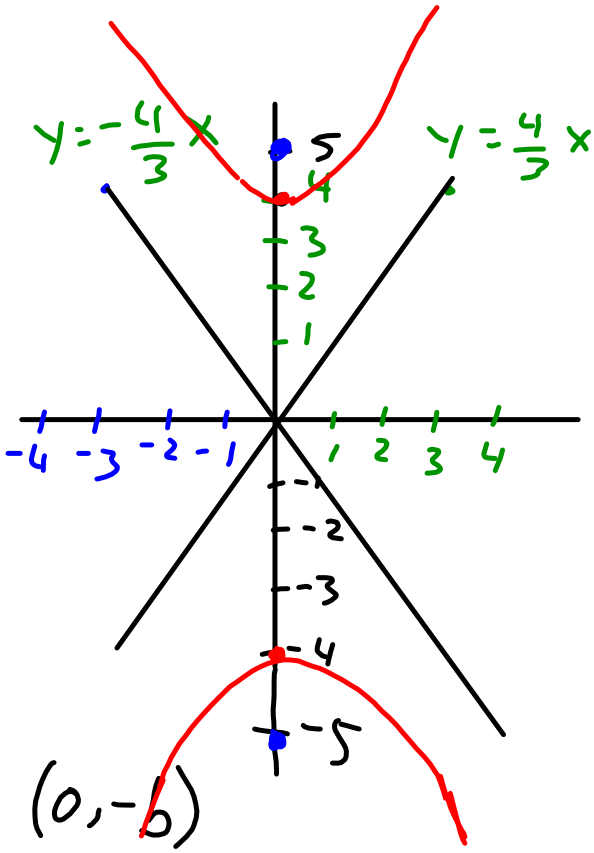
$c = 15$

Case 1 Example:



Case 2: Example

$$\frac{x^2}{3^2} - \frac{y^2}{4^2} = -1$$



① Sketch asymptotes.

$y = \frac{b}{a}x \quad y = \frac{-b}{a}x$

$y = \frac{4}{3}x \quad y = -\frac{4}{3}x$

② Vertices are $(0,b)$ $(0,-b)$

$(0,4)$ and $(0,-4)$

③ $c^2 = a^2 + b^2$

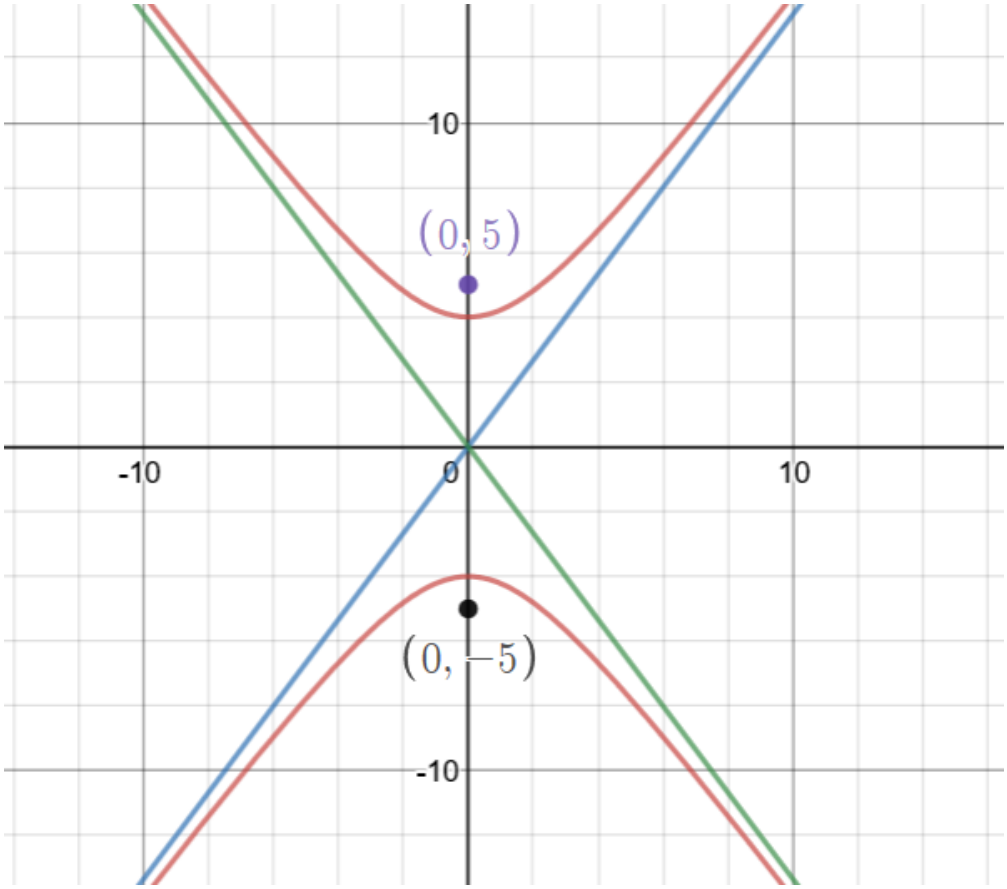
$c^2 = 3^2 + 4^2$

$c = 5$

$(0,5)$ $(0,-5)$

foci

Case 2 :

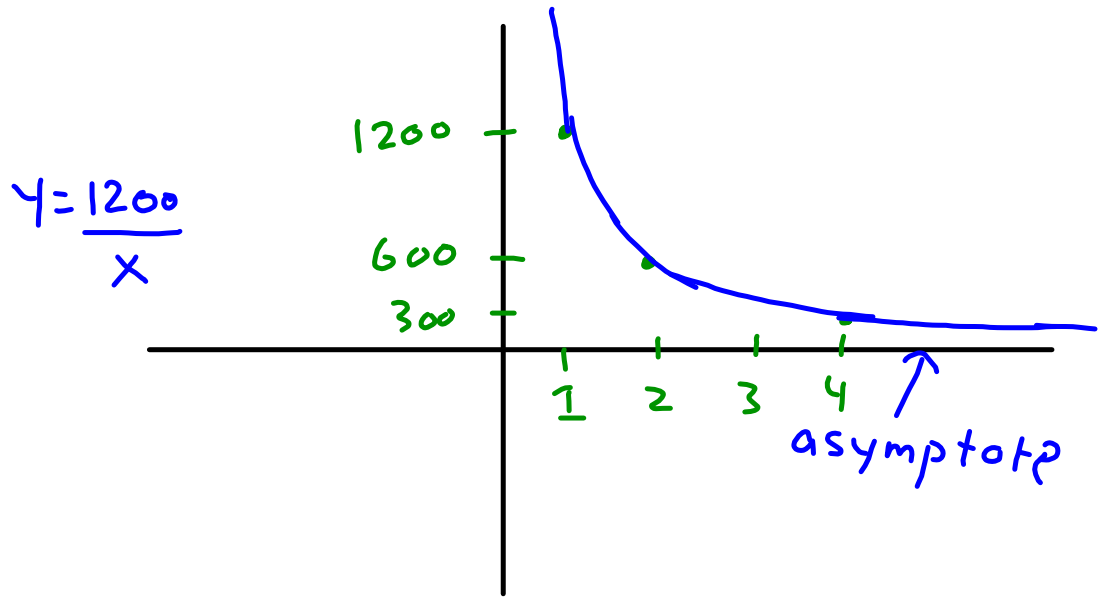


sketch and check on desmos

① $\frac{x^2}{4} - \frac{y^2}{16} = 1$

② $\frac{x^2}{3^2} - \frac{y^2}{5^2} = -1$

③ $\frac{x^2}{121} - \frac{y^2}{169} = -1$


y = 1200/x
1200
600
300
1
2
3
4
asymptote