Parabola

A parabola is a curve where all points are equidistant from a fixed line called the directrix, and a fixed point called the focus.

the equation of a parabola (standard form) is

Case 1:

$(x-h)^2 = 4c\,(y-k)$ (c is not equal to zero)

or

Case 2:

$(y-k)^2 = 4c(x-h)$ (c is not equal to zero)

- The coordinates of the vertex are (h,k)
- The distance between the focus and the directrix is $2|c|$

Case 1:

$(x-h)^2 = 4c\,(y-k)$ (c is not equal to zero)

> the equation of the parabola's axis of symmetry is x =h

> the coordinates of the focus are (h, k+c)

> the equation of the directrix is y = k-c

> The parabola opens upwards if c>0

> The parabola opens downwards if c<0

Ex : $(x-1)^2 = 2(y+3)$

$4c = 2$

$c = 0.5$

$F(1, -3+0.5)$

$F(1, -2.5)$

(h, k)

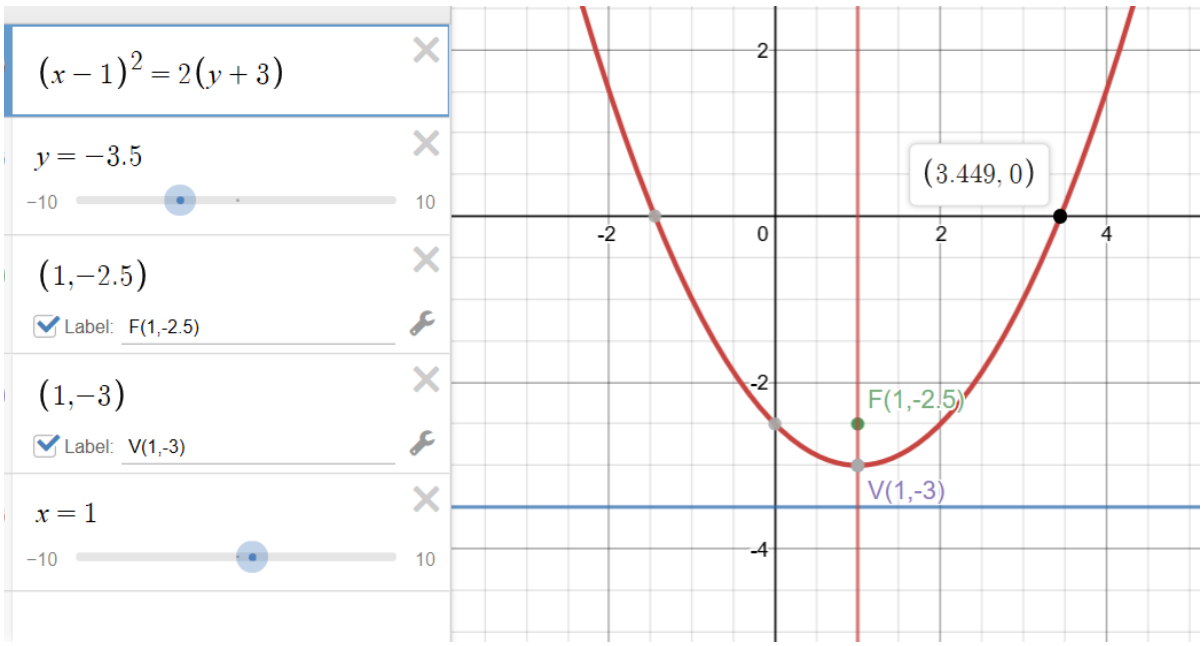
$(1, -3)$

Directrix: $y = k - c$

$y = -3 - 0.5$

$y = -3.5$

$c > 0$ opens upwards.



Case 2:

$(y-k)^2 = 4c(x-h)$ (c is not equal to zero)

› The equation of axis of symmetry is y = k

› The coordinates of the focus are (h+c, k)

› The equation of the directrix is x = h-c

› if (c<0), the parabola opens towards the left

› if (c>0), the parabola opens towards the right.

Ex:

$(y+2)^2 = -8(x-1)$

Observations:

$(h,k) :\rightarrow (1,-2)$

$4c = -8$

$c = -2$

opens ~~Right~~ Left

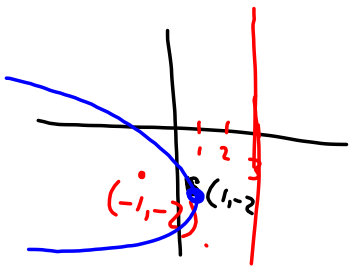
Equation of directrix: $x = h-c$

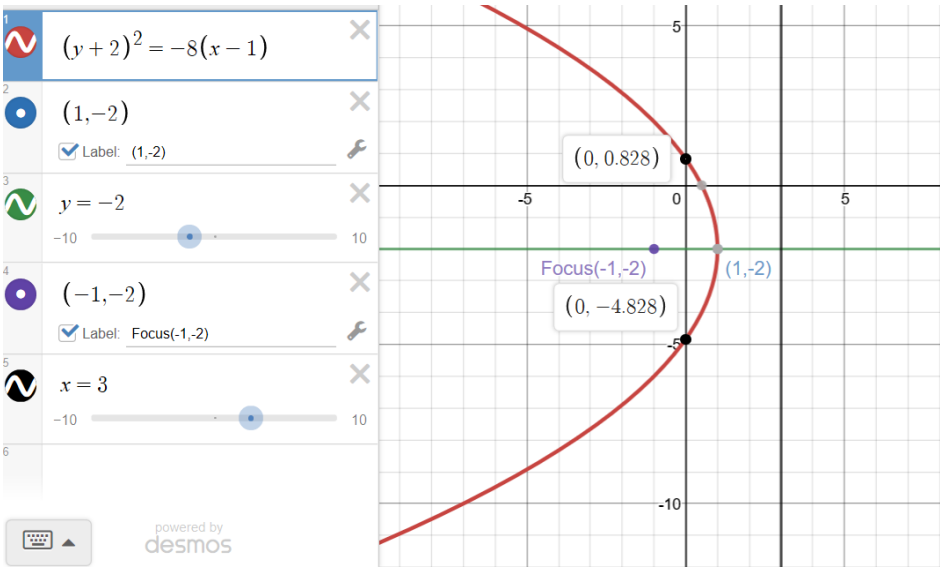
$x = 1-(-2)$

$x = 3$

$F(h+c, k)$

$(-1,-2)$





Work: Sketch the following and check your answers in Desmos

Q1:-

$(y-14)^2 = 6(x+8)$

Q2: $y^2 = -8x$

Q3: $x^2 = 2y$

Q4:- $(x-12)^2 = -8(y+2)$

Finding the equation of a parabola

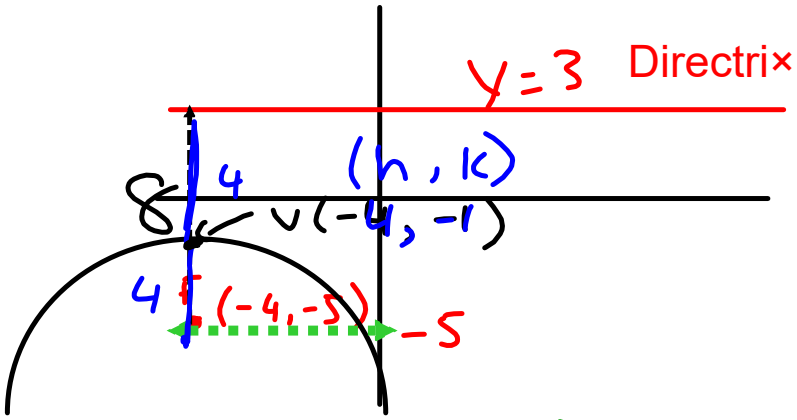
In order to find the equation of a parabola; Case 1 , or Case 2

follow the following steps:

› Deduce some information concerning parameters c, h and k

› Write the equation of the parabola

Ex1:

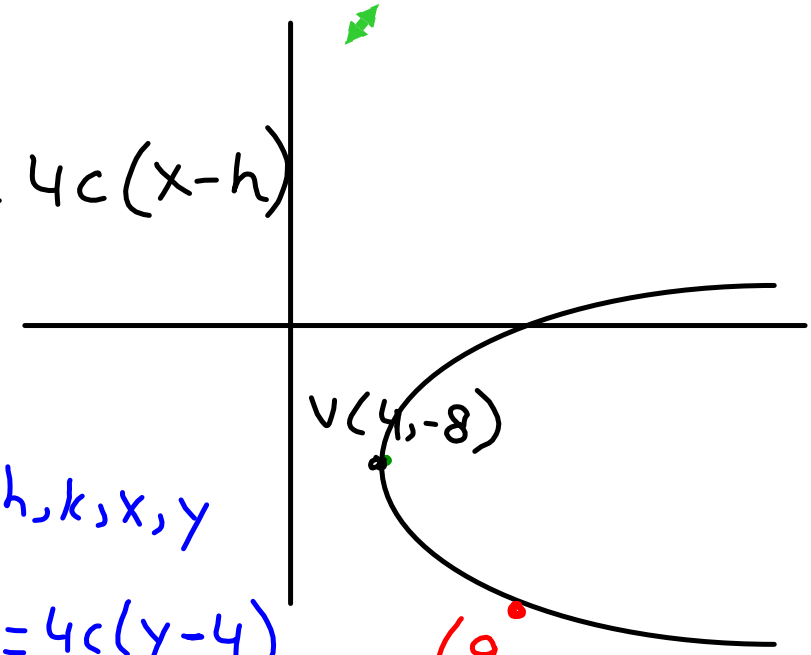


$8 = 2|c|$ $\quad (x-h)^2 = 4c(y-k)^2$

$c = -4$ $\quad (x+4)^2 = -16(y+1)$

Case 2:

$(y-k)^2 = 4c(x-h)$



$h = 4$

$k = -8$

Plug in h, k, x, y

$(-18-(-8))^2 = 4c(y-4)$

$(-18+8)^2 = 4c(9-4)$

$(-10)^2 = 4c(5)$

$100 = 20c$

$c = \frac{100}{20} = 5$

$(y-k)^2 = 4c(x-h)$

$(y+8)^2 = 20(x-4)$

Shading inequalities in a parabola

Ex: Graphically represent the region corresponding to the inequality : $(x+2)^2 < -0.5\,(y-3)$

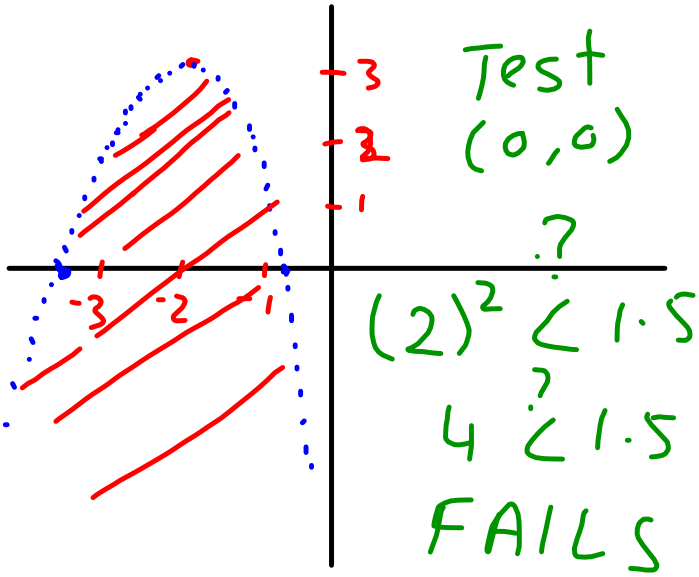
$(h,k) = (-2,3)$

When $y=0$

$(x+2)^2 = -0.5(0-3)$

$(x+2)^2 = (-0.5)(-3)$

$(x+2)^2 = 1.5$



$x+2 = 1.22$ | $x+2 = -1.22$

$x = 1.22 - 2$ | $x = -1.22 - 2$

$x = -0.78$ | $x = -3.22$

X-intercepts

$(-0.78, 0)$ | $(-3.22, 0)$

Work (WB).

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