**2- Correction key**

5

C

6

Work : (example)

Calculation of *w*

In 1960, *x* was 10 and f(*x*) = 3500 so that

|  |  |
| --- | --- |
| 3500 =  35 × 100 =  100 =  102 = | 35(1010*w*)  35(1010*w*)  1010*w*  1010*w* |

thus 2 = 10*w* i.e. *w* = 0.2

Result : The parameter is 0.2.

8

Work : (example)

f(*x*) = 200 × (1.04)5

f(*x*) = $243.33

Result : $243.33

9

Work : (example)





*f(x)* = 60 × 0.0077

*f(x)* = 0.46

Result : 0.46 grams

10

The rule that corresponds to function t is t(*n*) = 1225 × 1.06*n*

14

The rule is Q(*t*) = 100 × 32*t*

18

Example of an appropriate method

Equation of the function

|  |
| --- |
| The population doubles every 10 years |
|  |

Calculation of the population in the year 2000

|  |
| --- |
| P(*x*) = |
| P(*x*) = 130 000(2)2.5 |
| P(*x*) ≈ 735 391.05 |

Answer On January 1st 2000, there were 735 391 people in Kilwat.

20

Example of an appropriate method

Value of investment under option A after 2 years

*C*(*t*) = 2000 

*C*(*t*) = 2000 (1.05)*t*

*C*(*t*) = 2000 (1.05)2

= $2205

Rule associated with investment option B

*C*(*t*) = 2000 

*C*(*t*) = 2000 (1.0035)12*t*

Number of months required under investment option B to earn $2205

2205 = 2000(1.0035)12*t*

 = (1.0035)12*t*

1.1025 = (1.0035) 12*t*

12*t* = log1.0035 1.1025

12*t* = 27.9288…

Answer: Gerry would have had to invest his money for **28** months, to the nearest whole month, under investment option B in order to earn the same amount he will earn under investment option A

21

Example of an appropriate solution

Let *t*: time, in minutes, which has past since 21:00

*A*(*t*): altitude of the airplane after *t* minutes

The rule of correspondence

*A*(*t*) = *a* × *ct*



*t* when *A*(*t*) = 280



Answer: The airplane will be at an altitude of 280 m at **21:22**.

22

Example of an appropriate solution

N = 27.04 g NO = 100 g *t* = 3000 years





Answer: In 1000 years, there will be **161.66** g of radium left.

24

The message will have been sent to exactly one million people on **Sunday**.

25

Example of an appropriate solution

Let *C0*, be the original investment

*i*, the rate of interest

*n*, the number of interest payments per year

*t*, the length of the investment period



Answer: It will take Jeremy **17.67** years or **17 years and 8 months** to quadruple his investment.

Accept all answers between 17.67 and 18 years.

26

Example of an appropriate solution

Let *t*: time after 1995 (years)

V(*t*): value of the car ($)

V(*t*) = 17 500(*r*)*t* where *r* is the rate at which the value declines and

10 000 = 17 500(*r*)3

0.5714285 = (*r*)3

*r* ≈ 

*r* ≈ 0.83

When V(*t*) = 5000

5000 ≈ 17 500(0.83)*t*

≈ (0.83)*t*



*t* ≈ 6.72

Answer The value of the car falls below $5000 when it is 6.72 years ≈ 6 years 9 months.

29

Example of an appropriate method

Find the rate

|  |  |
| --- | --- |
| *y* = | a • bx |
| *y* = | 110 • bx |
| 835 = | 110 • bx |
| 7.59 = | b5 |
| 1.5 = | b |

Find time that elapsed when 2000 victims have been infected

110 × 1.5*t* = 2000

1.5*t* = 18.

*t* =  ≈ 7.15

Find the year the vaccine will be offered

1996 + 7.15 = 2003.15

Answer: The population will be offered the vaccine in the year **2003**.

30

Example of an appropriate solution

Let *t* be the number of years after 1990

|  |  |
| --- | --- |
| 5.5(1 + 0.019)*t* = | 9 |
| (1 + 0.019)*t* = |  |
| *t* log (1.019) = | log |
| *t* = | ≈ 26.165... |

Answer: The population will reach 9 billion in the year **2016**.

34

Example of an appropriate solution

Let *t*: number of days

*f*(*t*): amount of the compound remaining (g)



Time for 75 g to remain:



Answer: To the nearest day, half of the compound will remain after **34** days

35

Example of an appropriate solution

Mathematical model: *y = acx*

Determine base, *c* using ordered pairs (1865, 60 000 ) and (1867, 2 400 000)



Let *x* represent the number of years from the introduction of rabbits to 1865:



Year when first pair of rabbits was brought to Australia.

1865 − 5.589… = 1859.410…

Answer: The first pair of rabbits was brought to Australia in the year **1859**.

Name : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Group : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Date : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**568536 - Mathematics**

**Question Booklet**

5

After studying the evolution of a population of 2000 gulls, biologists concluded that the population increased by 15 % every two years.

If this rate of growth is maintained, which rule can be used to find the number N of gulls there will be in *t* years?

|  |  |  |  |
| --- | --- | --- | --- |
| A) |  | C) |  |
| B) |  | D) |  |

6

Computers have changed a great deal since 1950 because of the miniaturization of the circuits.

|  |  |
| --- | --- |
| Year | Number of circuits  on a chip  f(*x*) |
| 1950  1960  1970  1980  1990 | 35  3 500  350 000  35 000 000  .... |

The function representing this situation is given by

f(*x*) = 35(10*wx*)

where *x* represents the number of years since 1950 and *w*, a parameter.

What is the value of the parameter?

8

A basket of groceries today costs $200. If the rate of inflation remains at 4 % for the next few years, how much will the same grocery basket cost in 5 years?

Note : Express your answer to the nearest hundredth.

Show all the steps in your solution.

9

A radioactive substance disintegrates at a rate such that after 2 years it has  of its initial mass. If you have 60 grams of this substance, how much of it will remain after 12 years?

Note : Express your answer in grams to the nearest hundredth.

Show all the steps in your solution.

10

To cover the cost of building a water filtration plant, a municipality is planning an average tax increase of 6 % per year starting in 1994.

Mr. Blais paid $1225 in taxes for the year 1993. He wants to find a function t that can be used to calculate the amount of annual taxes as a function of the number of years n elapsed since 1993.

What rule corresponds to function t?

The rule that corresponds to function t is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

14

During an experiment in a science laboratory, a population of 100 mosquitoes triples every 12 hours.

What rule can be used to calculate the number of mosquitoes Q(*t*) as a function of the number of days *t*?

The rule is Q(*t*) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

18

The number of people living in Kilwat, Germany, varies according to the rule of an exponential function. On January 1st 1975, the city's population was 130 000. On January 1st 1985, it was 260 000.

What was the population of this German city on January 1st 2000, given that the growth rate remained constant?

Show all your work.

20

When interest is paid n times a year, the value of a certain amount of capital C0 invested at an annual interest rate i for *t* years will be as follows:

*C*(*t*) = C0

Gerry wants to invest $2 000 for 2 years. He has two investment options.

|  |  |  |
| --- | --- | --- |
| Investment option A  Annual interest rate of 5%  Interest paid once a year |  | Investment option B  Annual interest rate of 4.2%  Interest paid 12 times a year |

Gerry chose investment option A because he was told it would provide the best return.

Rounded to the nearest whole month, how many months would Gerry have had to invest his money under investment option B in order to earn the same amount he will earn under investment option A?

Show all your work.

21

An airplane is flying at an altitude of 10 000 m. At 21:00, the pilot begins the descent towards Pierre Elliott Trudeau Airport. The descent follows an exponential model ending with the plane’s landing. At 21:04, the airplane is at an altitude of 5222 m.

At what time will the airplane be at an altitude of 280 m?

Show all your work.

22

Half-life is the time required for a radioactive element to decay to half its original mass.

Radium (Ra) is a radioactive element that decays naturally according to the following equation:



where N is the mass in grams left after *t* year(s),

NO is the original mass in grams, and *d* is the half-life of the element.

In 3000 years, a 100-g sample of radium decays to a mass of 27.04 g.

In 1000 years, how much of a 250-g sample of radium will be left?

Show all your work.

24

Electronic mail makes it easy to contact many people rapidly. Justin received a message on Monday. The next day he sent it to 12 people. The following day, each of those 12 recipients sent the message to 12 people, and so on.

On what day will the message have been sent exactly one million times?

The message will have been sent to exactly one million people on \_\_\_\_\_\_\_\_\_.

25

Jeremy inherited $80 000 from an aunt. He wants to invest this money for his retirement needs. One bank offers him an annual rate of 8% paid out twice a year.

How long will it take Jeremy to quadruple his original investment?

Show all your work.

26

When Jennifer bought a new car in 1995, she paid $17 500. In 1998 the value of her car had fallen to $10 000. She decided that she would sell her car when the value fell below $5000.

Assuming the decline in the price of a car is modelled by an exponential function, how old will Jennifer's car be when its value falls below $5000? Round your answer to the nearest month.

Show all your work.

29

A virus appeared in South America in the middle of the last decade. Scientists knew that the number of people infected with this virus would increase according to a specific exponential function.

At the beginning of 1996, authorities found 110 infected people. Five years later, the number had grown to 835. Wide-scale inoculation began once 2000 people had been infected with the virus.

In what year did these inoculations begin?

Show all your work.

30

In January 1990, there were 5.5 billion people living on this planet. The population has been growing at a rate of 1.9% per year.

In which year will the population reach 9 billion?

Show all your work.

34

|  |  |
| --- | --- |
| A chemist is working with a dangerous compound she has just created. She began with 150 g of the compound, but noticed that it decays exponentially. After observing for 10 days, 123 g remained. She needs to know how long it will take until only half of the compound will be left.  Rounded to the nearest day, how many days after the experiment started will only half of the compound remains?  Show all your work. |  |

35

When rabbits were first brought to Australia, they had no natural enemies. From January 1865 to January 1867, the rabbit population increased exponentially from 60 000 members to 2 400 000 members.

According to this exponential model, in which year were the first pair of rabbits brought to Australia?