Hyperbola

Definition: A hyperbola is a curve where the absolute value of the difference of the distance between any point on the curve and two fixed points, called foci, is constant.

Equation:

The equation of a hyperbola is in the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \pm 1$$

> The coordinates of the center are (0,0)

> The lines associated with the equations: $y = \frac{b}{a}x$, $y = \frac{-b}{a}x$ are the asymptotes of the curve

> The relationship among the values of the parameters a,b and the distance c (between the center of hyperbola and one of its foci) is represented by

$$c^2 = a^2 + b^2$$

if the equation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (case 1)

> The coordinates of the vertices are (a,0) and (-a,0)

> The foci are located on the x-axis and their coordinates are (c,0) and (-c,0)

> The absolute value of the difference of a point on the curve and the two foci is 2a $|d(M,F_1) - d(M,F_2)| = 2a$

if the equation is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ (case 2)

> The coordinates of the vertices are (0,b) and (0,-b)

> The foci are located on the y-axis, and their coordinates are (0,c) and (0,-c)

> The absolute value of the difference of a point on the curve and the two foci is 2b : $|d(M,F_1) - d(M,F_2)| = 2b$

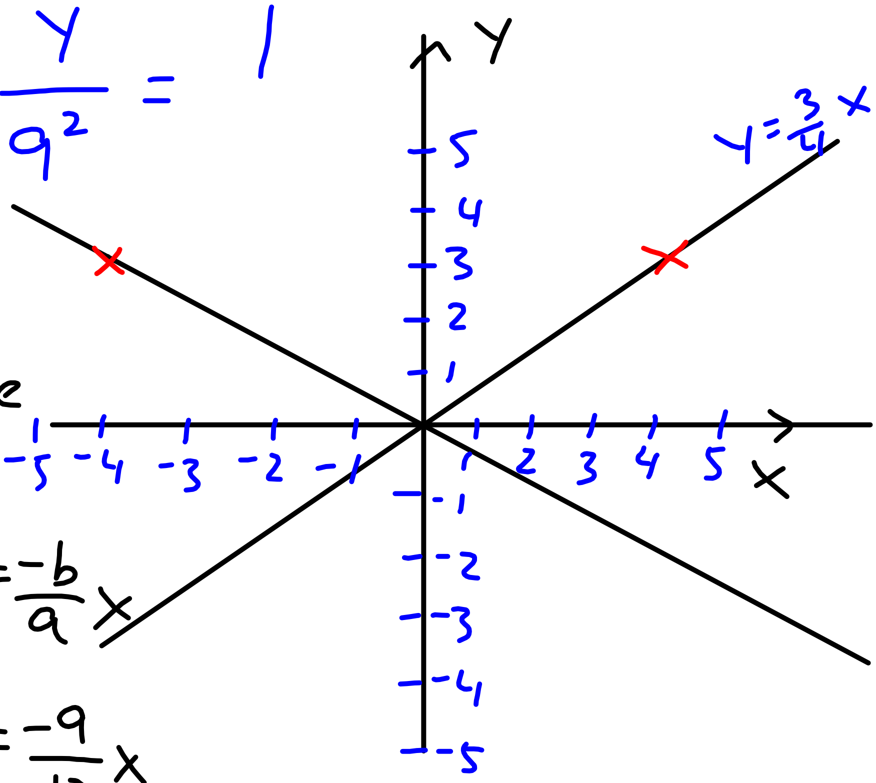
Case 1:

$$\frac{x^2}{12^2} - \frac{y^2}{9^2} = 1$$

sketch:

① sketch the asymptotes

$y = \frac{b}{a}x$ and $y = \frac{-b}{a}x$



$y = \frac{9}{12}x$ $y = \frac{-9}{12}x$

$y = \frac{3}{4}x$, $y = \frac{-3}{4}x$

Coordinates of the vertices.

(12, 0) and (−12, 0)

Coordinate of the foci

(15, 0) (−15, 0)

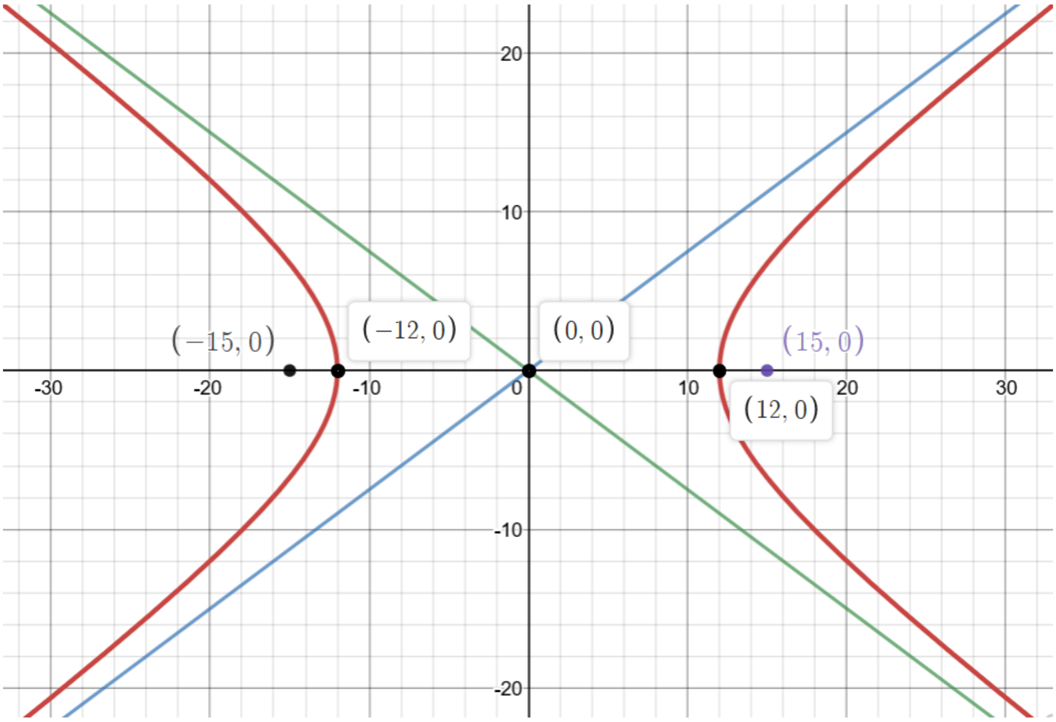
$c^2 = a^2 + b^2$

$c^2 = 12^2 + 9^2$

$c = \sqrt{225}$

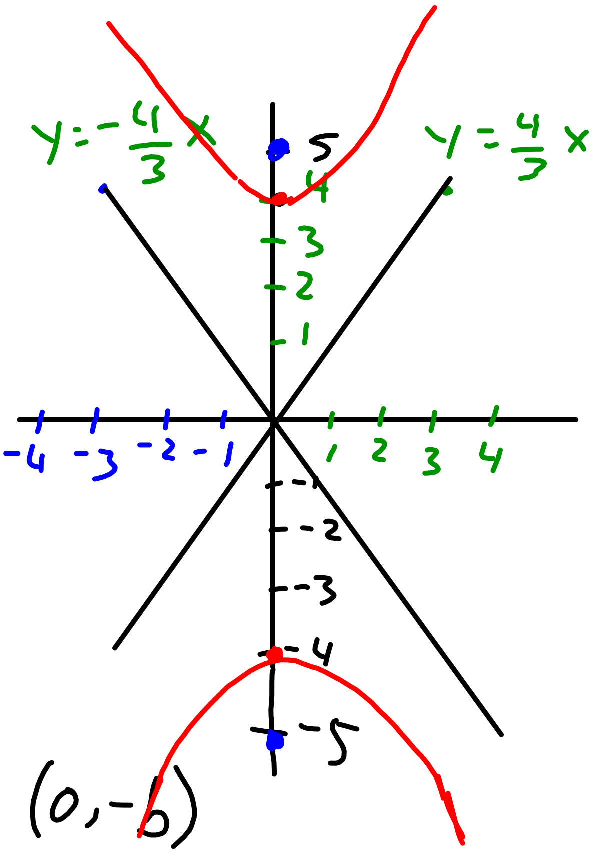
$c = 15$

Case 1 Example:



Case 2: Example

$$\frac{x^2}{3^2} - \frac{y^2}{4^2} = -1$$



① Sketch asymptotes.

$y = \frac{b}{a}x \quad y = \frac{-b}{a}x$

$y = \frac{4}{3}x \quad y = -\frac{4}{3}x$

② Vertices are $(0,b)$ $(0,-b)$

$(0,4)$ and $(0,-4)$

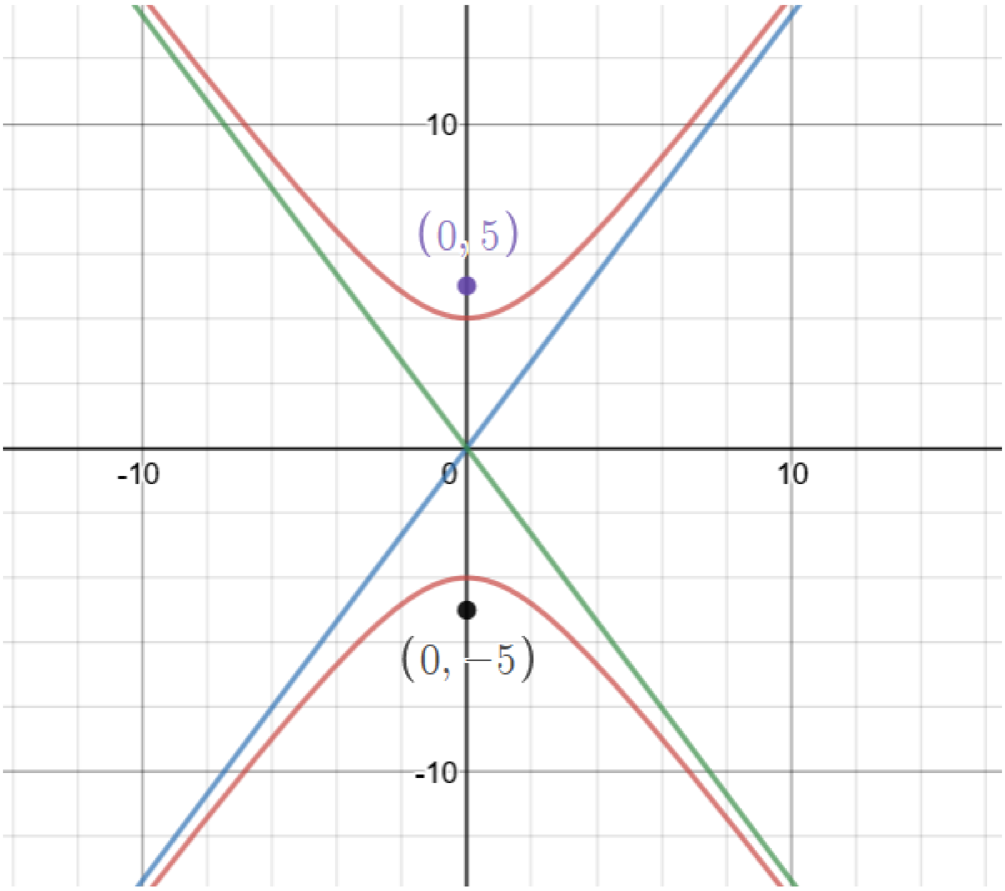
③ $c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c = 5$

$\underbrace{(0,5) \quad (0,-5)}_{\text{foci}}$

Case 2 :



sketch and check on desmos

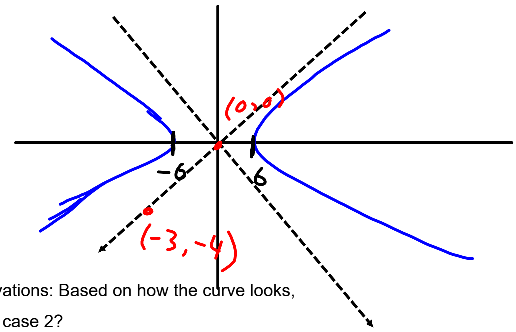
1. $\dfrac{x^2}{4} - \dfrac{y^2}{16} = 1$

2. $\dfrac{x^2}{3^2} - \dfrac{y^2}{5^2} = -1$

3. $\dfrac{x^2}{121} - \dfrac{y^2}{169} = -1$

Finding the equation of a hyperbola

Ex 1: Find the equation of the given hyperbola



Some observations: Based on how the curve looks, is it case1 or case 2?

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since we know two points on the asymptote, we can find the equation of both asymptotes.

$x_1 \quad y_1$
$(-3, -4)$
$(0, 0)$
$x_2 \quad y_2$

slope: $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{0 - -4}{0 - -3} = \frac{4}{3}$$

$$\boxed{y = \frac{4}{3}x} \qquad \boxed{y = -\frac{4}{3}x}$$

Recall that the equations of the asymptotes are

$$\boxed{y = \frac{b}{a}x} \qquad \boxed{y = \frac{-b}{a}x}$$

From the Graph $a = 6$ (coordinate of the vertices)

$$\frac{b}{6} = \frac{4}{3}$$

$$b = \frac{4 \times 6}{3} = 8$$

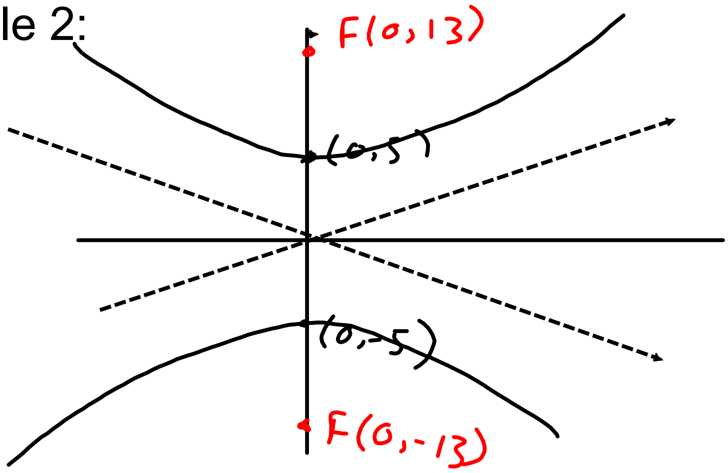
$\therefore \quad a = 6, \; b = 8$

Equation

$$\frac{x^2}{6^2} - \frac{y^2}{8^2} = 1$$

OR $$\frac{x^2}{36} - \frac{y^2}{64} = 1$$

Example 2:



Case 2:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$$

b is Given = 5

C is Given

$c=13$

$c^2=a^2+b^2$

$13^2=a^2+5^2$

$169-25=a^2$

$144=a^2$

$\sqrt{144}=a=12$

$a=12 \quad b=5$

Equation

$$\frac{x^2}{12^2}-\frac{y^2}{5^2}=-1$$

$$\frac{x^2}{144}-\frac{y^2}{25}=-1$$

inequality shading.

Graph the solution to the following.

$$9x^2 - 4y^2 \le 36$$

Divide by 36

$$\frac{9x^2}{36} - \frac{4y^2}{36} \le \frac{36}{36}$$

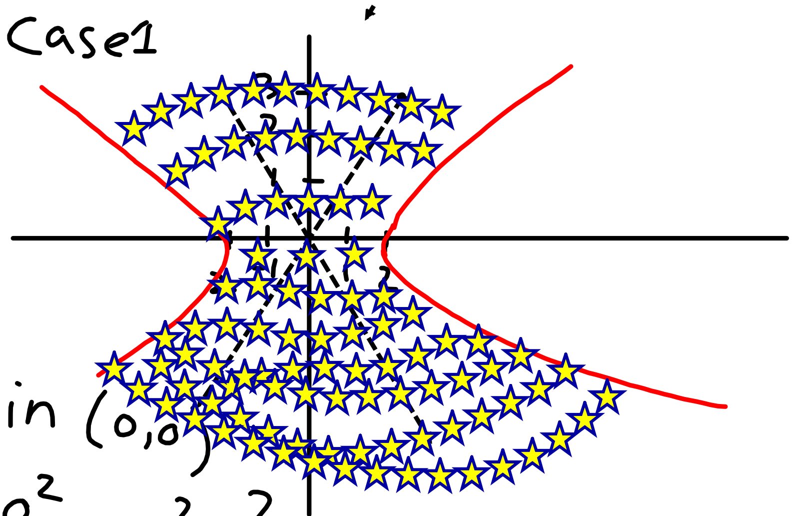
$$\frac{x^2}{4} - \frac{y^2}{9} \le 1$$

OR $\frac{x^2}{2^2} - \frac{y^2}{3^2} \le 1$

$y = \frac{3}{2}x$

$y = -\frac{3}{2}x$

Case1



Plug in (0,0)

$$= \frac{0^2}{2^2} - \frac{0^2}{3^2} \overset{?}{\le} 1$$

$$0 \overset{?}{\le} 1$$

Yes.

P 338- 343

Q 1- 20

$y = \frac{1200}{x}$