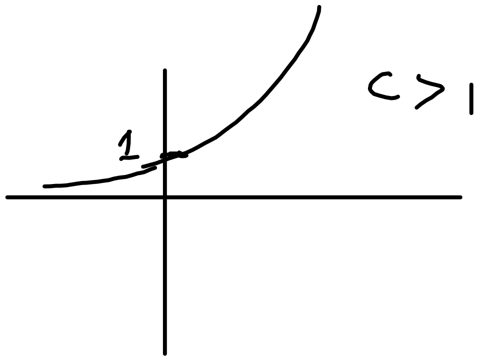
Exponential Function $f(x) = c^x$

The function $f(x) = c^x$, where c is a positive real number different from 1, is called the basic exponential function with 'c' as its base.

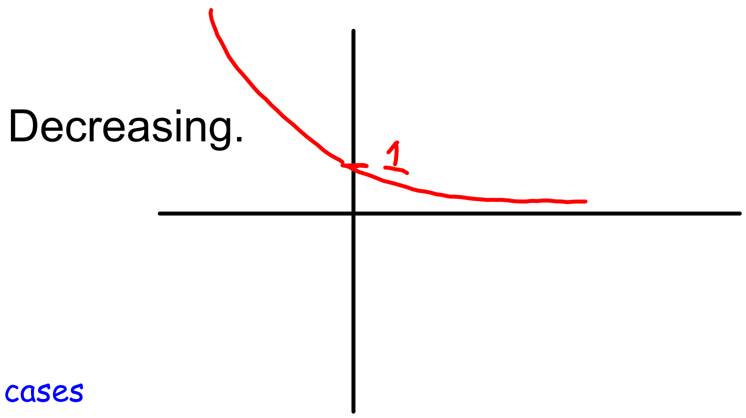
Depending on the value of the base, it can be a function of growth or decay.

Case 1: If c > 1



increasing function (growth)

Case 2: if 0<c<1, then it is a decreasing function.



For both cases

- dom f = R (all real numbers) and range f = $R^+ - \{0\}$ (positive real numbers excluding zero)
- The initial value is equal to 1 as ($c^0=1$)
- The function has no zero
- The function is positive over R
- The x-axis is an asymptote to the curve

<u>The most common Bases</u>

The most common bases of the exponential function are the numbers 10 and e.

e, like pi, is an irrational number commonly embedded in how nature is designed. It is closely linked to the golden ration

e= 2.71828.....

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Q4:-

$(-2, \overset{?}{y})$

$A\left(-\frac{1}{2}, \frac{2}{3}\right) \in y = c^x$

$\frac{2}{3} = c^{-\frac{1}{2}}$

$\frac{2}{3} = \frac{c^{-\frac{1}{2}}}{1}$

$\frac{2}{3} = \frac{1}{c^{1/2}}$ (flip both sides)

$\frac{3}{2} = c^{1/2}$ (square both sides)

$\left(\frac{3}{2}\right)^2 = \left(c^{1/2}\right)^2$

$\boxed{\frac{9}{4} = c}$

Rule

$y = \left(\frac{9}{4}\right)^x$

(a) Plug in $x = -2$

$y = \left(\frac{9}{4}\right)^{-2} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

(b) Plug in $y = \frac{4}{9}$

$y = \left(\frac{9}{4}\right)^x$

$\left(\frac{4}{9}\right)^1 = \left(\frac{9}{4}\right)^x$

$\left(\frac{9}{4}\right)^{-1} = \left(\frac{9}{4}\right)^x$

therefor $x = -1$

p139-140

Q1-6

Exponential Function

A function defined by a rule in which the independent variable (x) appears as an exponent.

The rule of an exponential function is $y = ac^{b(x-h)} + k$ where

a and b are NOT equal to zero

the base c is greater than 0 and not equal to 1

Using the laws of exponents you can transfer this rule to the standard form, which is $f(x) = ac^x + k$

Ex:

$$f(x) = 5(3)^{2(x+1)} + 7$$

Recall: $\boxed{a^m \cdot a^n = a^{m+n}}$

$$f(x) = 5(3)^{2x+2} + 7$$

$$= 5(3)^{2x}(3)^2 + 7$$

$$5(9)(3)^{2x} + 7$$

$$45(3)^{2x} + 7$$

Law

$$45(3^2)^x + 7 \qquad (a^m)^n = a^{mn}$$

$$f(x) = 45(9)^x + 7 \quad \text{Standard.}$$

Graphing the exponential Function

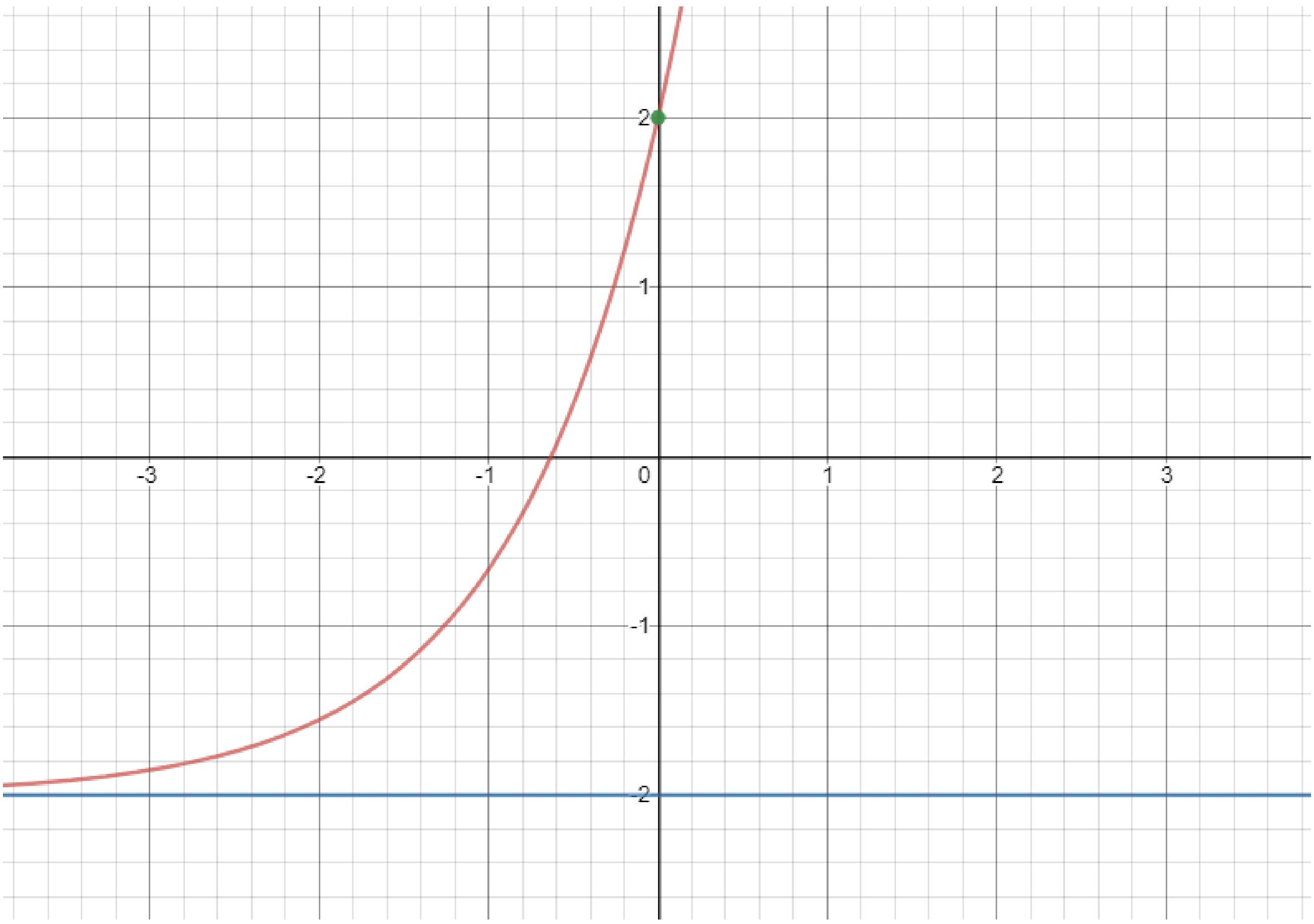
While graphing $f(x) = ac^x + k$, the curve passes through the point with coordinates (0, a+k), and one of its ends approaches a horizontal asymptote represented by the equation y = k

Ex: Rule $y = 4(3)^x - 2$

> increasing function as c >1

> Passes through the point (0, 4 + (-2)) i.e. (0,2)

> asymptote y = -2

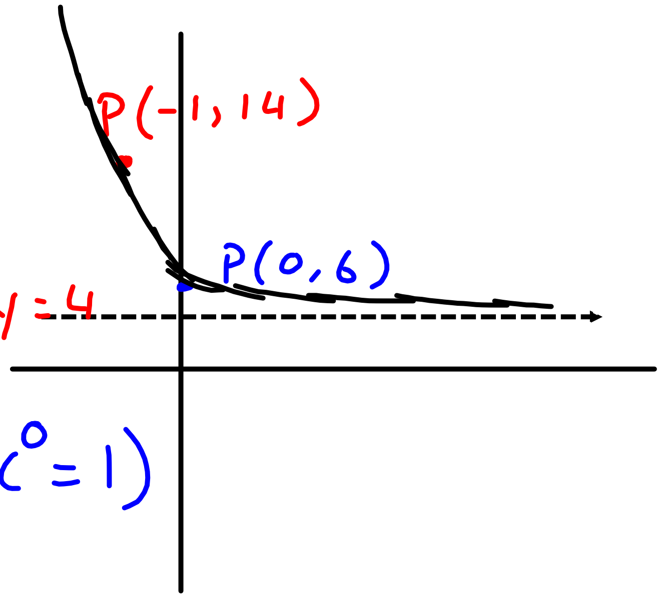


Finding the Rule of an exponential function

$y = ac^x + k$

$y = ac^x + 4$

Plug in $(0, 6)$



$6 = ac^0 + 4 \quad (c^0 = 1)$

$6 = a + 4$

$a = 6 - 4$

$\boxed{a = 2}$

$y = 2c^x + 4$ (Plug in $(-1, 14)$)

$14 = 2c^{-1} + 4$

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

$14 = \frac{2}{c} + 4$

$14 - 4 = \frac{2}{c}$

$10 = \frac{2}{c} \Rightarrow 10c = 2$

$c = \frac{2}{10} = \frac{1}{5} = 0.2$

Rule: $y = 2(0.2)^x + 4$

Questions

Textbook V1.

p173, 174

Q2, 3, 4, 5,

p173

Q4:-

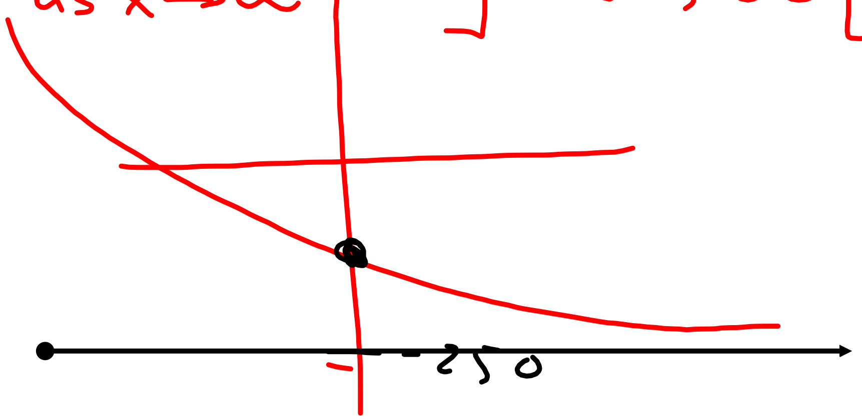
(a) $f(x) = 2\left(\frac{1}{5}\right)^{x+8} - 250$

$2\left(\frac{1}{5}\right)^{x}\left(\frac{1}{5}\right)^{8} - 250$

$0.00000512\left(\frac{1}{5}\right)^{x} - 250$

Domain: $\mathbb{R}$

$f(x) \searrow$ as $x \to \infty$ $]-250, \infty[$



Exponential Equation - Form $c^u = c^v$

When both sides of an exponential equation can be written as powers of the same base, we use the logical equivalence

$c^u = c^v \Leftrightarrow u = v$

Key step: Makes bases on both sides equal. It follows that the exponents are equal as well.

$$5(2)^{3x} = 320$$

Divide by 5

$$2^{3x} = 64$$

Notice $64 = 2^6$

$$2^{3x} = 2^6$$

So $3x = 6$

$$x = \frac{6}{3} = \boxed{2}$$

Rule of an Exponential Function $f(x) = ac^{bx}$

Once the unit of time is well defined, x represents the elapsed time. y could be the number of bacteria, money gained etc

a= initial value

b = the number of periods per unit time

c = is the keep (in case of percentage gain, calculate it by using

1+percentage , or 1- percentage); in case of bacteria doubling or tripling c would be 2 and 3 respectively.

Ex: In a controlled environment, there are initially 10 bacteria. The unit of time is in hours.

- When the number of bacteria triples (c=3) every 15 minutes, the rule will be $y = 10(3)^{4x}$. (why the 4?)

- When the number of bacteria double every 30 minutes, the rule is

$y = 10(2)^{2x}$

- When the number of bacteria quadruples every 2 hours, the rule is $y=10(4)^{0.5x}$

p 149,150

Q14, Q15, 16,18,19

p 152 Q20, Q24(try it)

Q25,26,27

P 154 Q28,29,