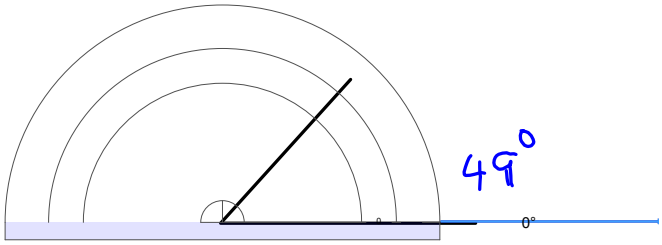
Measure of Angles

So far, we have measuring angles in degrees like 60 , 45, 30 degrees



Another measure of angle is called radians!

Radians are much bigger than degrees so big angles can be expressed very concisely.

π radians $= 180^\circ$

1 radian $= \frac{180^\circ}{\pi}$

Q: How many Degrees in 3 Radians.

Ans: $3 \times \frac{180}{\pi} = \frac{540^\circ}{\pi}$

Q: How many Degrees in $\frac{\pi}{6}$ Radians?

$\frac{\pi}{6} \cdot \frac{180}{\pi} = 30^\circ$

Q: How many Degrees in

$\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5}{4}\pi, \frac{7}{2}\pi, 2\pi$

Radians.

$\frac{\pi}{7}$ Rad $=$ _____ deg.

$\frac{\pi}{7} \cdot \frac{180}{\pi} = \frac{180}{7} = 25.71^\circ$

Lesson 2

$180^\circ = \pi$ Rad

Half a circle = pi radians

A full circle or 360 degrees = 2pi radians

To change radian to degrees multiply the radian angle by $\frac{180}{\pi}$

Ex: $\frac{\pi}{2}$ Rad = ____ degrees.

$\frac{\pi}{2} \cdot \frac{180}{\pi} = 90^\circ$

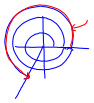
π Rad $= \pi \cdot \frac{180}{\pi} = 180^\circ$

2π Rad $= 2\pi \cdot \frac{180}{\pi} = 360^\circ$

Find the measure of the following angles in Degrees.

$\frac{\pi}{3}$ / $\frac{\pi}{4}$ / $\frac{3\pi}{4}$ / $\frac{4\pi}{3}$ / $\frac{7\pi}{4}$ / $\frac{16\pi}{3}$

60° / 45° / 135° / 240° / 315° / 960°



To convert an angle from degrees to radians, multiply by $\frac{\pi}{180}$

Ex: Re-convert from Deg to Rad

$60^\circ \longrightarrow \frac{60}{1} \cdot \frac{\pi}{180} = \frac{\pi}{3}$

$960^\circ = 960 \cdot \frac{\pi}{180} = \frac{16\pi}{3}$

improper $\longrightarrow$ Mixed fraction.

$\frac{16}{3} \longrightarrow 3\overline{)16}$ R 1 $\qquad 5\frac{1}{3}$

$5\frac{1}{3} = \frac{(5 \times 3) + 1}{3} = \frac{15+1}{3} = \frac{16}{3}$

$\frac{16\pi}{3}$ Rad $= 5\frac{1}{3}\pi$

$= (2\pi) + (2\pi) + 1\frac{1}{3}\pi$ Rad.

$5\frac{1}{3}\pi = 1\frac{1}{3}\pi = \frac{4\pi}{3}$

$\frac{4\pi}{3} \cdot \frac{180}{\pi} = 240^\circ$

Co-terminal angle is the actual effective angle measures from the zero position AFTER taking out all the extra circles!

A circle in degrees corresponds to 360 deg while in radians, it corresponds to 2π Rad

Find the co-terminal angle for the following

1. 750 deg
2. $\frac{19\pi}{3}$ rad
3. 1024 deg
4. $101\frac{1}{2}\pi$ Rad

A negative angle goes clockwise from the zero position. For example -30°

effectively corresponds to 330 deg as positive angles must be measured anti-clockwise.

What is the co-terminal angle for the following.

1) -750°
2) $-\frac{31\pi}{4}$ Rad.
3) -365°
4) $-101\frac{1}{2}\pi$ Rad.

$-1\frac{1}{2}\pi + 2\pi$

$= \frac{\pi}{2}$

To determine the co-terminal angle (in radians).

1. Convert the angle into a mixed fraction.
2. take out all the extra circles (i.e. multiples of 2π)
3. Write down what is left. If it is negative, add 2π to get the positive angle equivalence.

Ex: -31π $\qquad 4\overline{)31}$ R 7

Find the co-terminal angle represented by the following situations.

1.) $2\frac{4}{5}\pi \quad -2\frac{3}{5}\pi + \frac{7\pi}{10} \quad -2\frac{107}{5}\pi$

$\frac{8}{10}\pi - \frac{6\pi}{10} + \frac{7\pi}{10} - \frac{2147\pi}{10}$.

$= \frac{-205}{10}\pi \quad = -20\frac{5}{10}\pi$

$\frac{3\pi}{2}$

$\frac{-\pi}{2}$

2) $\frac{7}{11}\pi + 1\frac{9}{11}\pi - \frac{245}{11}\pi$

$\frac{7\pi}{11} + \frac{20\pi}{11} - \frac{245\pi}{11}$

$\frac{-218\pi}{11} = -19\frac{9}{11}\pi$.

$-1\frac{9}{11}\pi + 2\pi$

$\frac{2}{11}\pi$

Find the co-terminal angle

① $\frac{3\pi}{7} + \frac{206}{7}\pi + \frac{2}{7}\pi$

② $-\frac{3}{11}\pi - \frac{7}{11}\pi - \frac{214}{11}\pi + \pi$

Find the co-terminal angle

① $\frac{3\pi}{7} + \frac{206}{7}\pi + \frac{2}{7}\pi$

$\frac{211}{7}\pi = 30\frac{1}{7}\pi$

$7\overline{)211}$ 30, 210, 1

$\boxed{\frac{1}{7}\pi}$

② $\boxed{-\frac{3}{11}\pi - \frac{7}{11}\pi - \frac{214}{11}\pi} + \pi$

$11\overline{)224}$ 20, 220, 4

$\frac{-224\pi}{11} = -20\frac{4}{11}\pi$

$= -\frac{4\pi}{11} + \frac{2\pi}{1}$

$-\frac{4\pi}{11} + \frac{22\pi}{11} = \boxed{\frac{18\pi}{11}}$

$1\frac{7}{11}\pi + \pi$

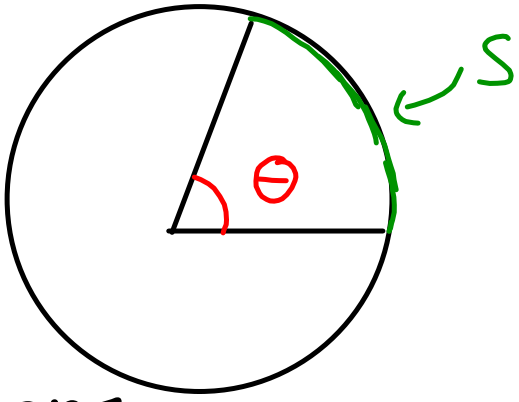
$2\frac{7}{11}\pi$

$= \boxed{\frac{7\pi}{11}}$

Arc Length

1. To find the arc length use the following formula

$S = r\theta$



The angle

must be in radians

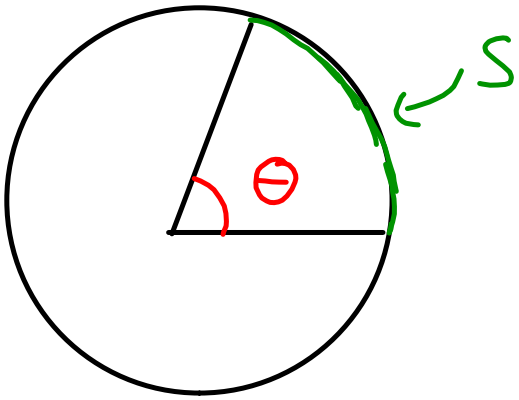
Find s if

$r = 3cm$

$\theta = 60°$

$\theta = 60 \times \frac{\pi}{180}$

$\theta = \frac{\pi}{3}$ Rad.



$S = r\theta$

$S = 3 \cdot \frac{\pi}{3}$

$S = \pi cm$

P196.

$s = r\theta$

$r = \frac{s}{\theta}$

$5\pi = r \cdot \frac{5\pi}{6}$

$5\pi \div \frac{5\pi}{6}$

$1 \times 6 = r$

$r = 6$

$5\pi \cdot \frac{6}{5\pi}$

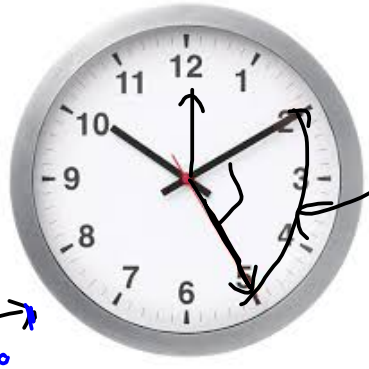
$= 6$

Q10:-

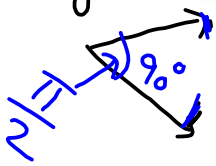
2:15 h

2 x circumf.

+ arc length



$r\left(\frac{\pi}{2}\right)$



$2\frac{1}{2}$

$C = 2\pi r$

$= 2\pi(0.8) = 1.6\pi$

$+ \quad S = r\theta$

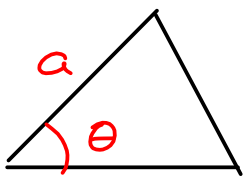
$S = 0.8\left(\frac{\pi}{2}\right) = 0.4\pi$.

2π cm.

P196.

Q7-12

Q12:-



$A = \frac{1}{2}ab\sin\theta$