SINUSOIDAL FUNCTION

A sinusoidal function is a periodic function whose rule can be written in the form $f(x) = a\sin b(x - h) + k$ or $f(x) = a\cos b(x - h) + k$ where $a \neq 0$ and $b \neq 0$. For a sinusoidal function, the following can be noted:

- The amplitude A is determined by $\frac{\max f - \min f}{2}$ and corresponds to the absolute value of parameter a.
- Period p is determined by $\frac{2\pi}{|b|}$.
- A cycle graphically corresponds to the smallest portion of the curve associated with the pattern that is being repeated.

> When the curve of a function is continuous, the point that makes the transition between one part of the curve that climbs (descends) more and more sharply and another part of the curve that climbs (descends) less and less sharply or vice versa is a **point of inflection**.

In the graphical representation of a sine function whose rule is written as $f(x) = a\sin b(x - h) + k$, (h, k) are the coordinates of a point of inflection of the curve.

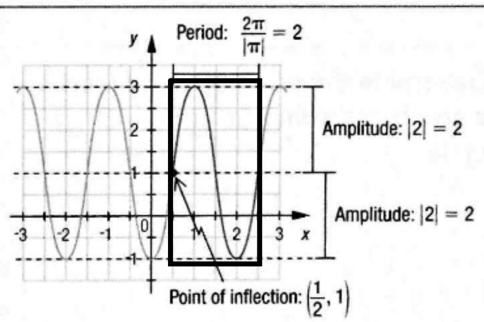
E.g.

Rule: $f(x) = 2\sin\pi\left(x - \frac{1}{2}\right) + 1$

Table of values

x	y
-3	3
-2	-1
-1	3
0	-1
1	3
2	-1
3	3

Graphical representation



The red portion of the curve corresponds to a cycle of the function.

In the graphical representation of a cosine function whose rule is written $f(x) = a\cos b(x - h) + k$, (h, k ± a) are the coordinates of a point associated with an extremum of the function.

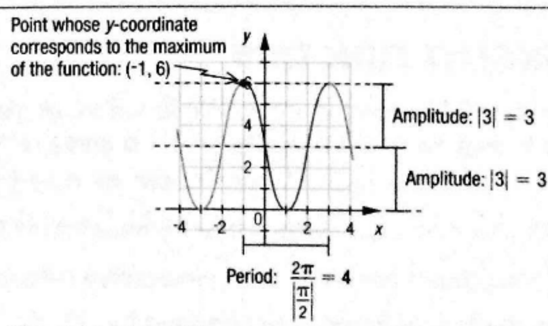
E.g.

Rule: $f(x) = 3\cos\frac{\pi}{2}(x + 1) + 3$

Table of values

x	y
-3	3
-2	-1
-1	3
0	-1
1	3
2	-1
3	3

Graphical representation

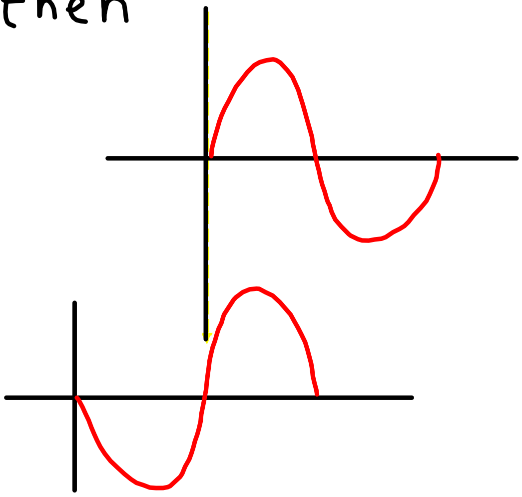


The red portion of the curve corresponds to a cycle of the function.

$a \sin b(x-h) + K$

if $ab > 0$, then

if $ab < 0$



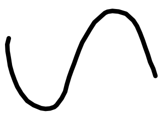
Ex:

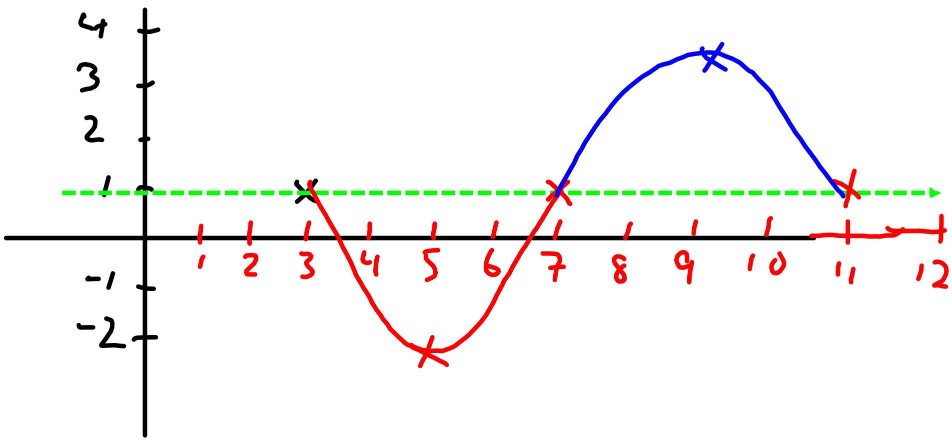
$$-2\sin\frac{\pi}{4}(x-3)+1$$

1) Amplitude $= |-2| = 2$

2) Period: $\frac{2\pi}{|b|} = \frac{2\pi}{\left|\frac{\pi}{4}\right|} = 8$

3) $(h, k) :\longrightarrow (3, 1)$

4) $ab < 0$: 

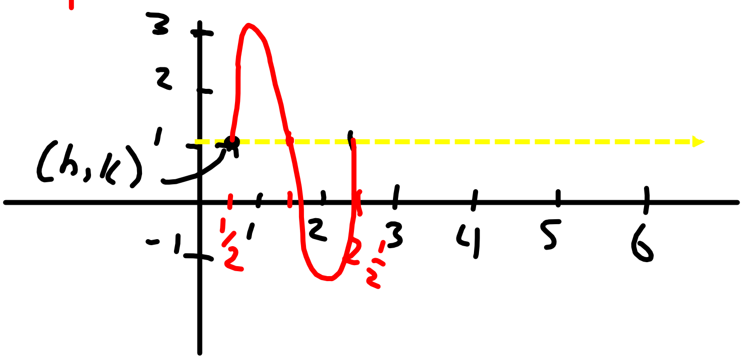


$$2\sin\pi\left(x-\tfrac{1}{2}\right)+1$$

$$(h,k) = \left(\tfrac{1}{2}, 1\right)$$

$$p = \frac{2\pi}{|b|} = \frac{2\pi}{|\pi|} = \frac{2\pi}{\pi} = 2$$

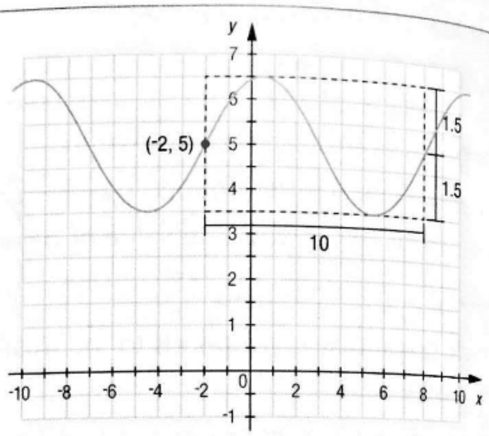
$$\text{amp} = 2$$



FINDING THE RULE OF A SINUSOIDAL FUNCTION

You can determine the rule of a sinusoidal function, whose rule is written $f(x) = a\sin b(x - h) + k$, or $f(x) = a\cos b(x - h) + k$, as follows:

$f(x) = a\sin b(x-h)+k$

1. Identify a cycle of the function whose starting point is associated with parameters **h** and **k**, and define this cycle using a rectangle whose base corresponds to period *p* and whose height is twice the amplitude *A*.	E.g.  By considering that the rule you want to find is that of a sine function, the coordinates of the starting point of the identified cycle are (-2, 5), the period is 10, and the amplitude is 1.5. $b = \frac{2\pi}{p}$
2. Determine the value of parameters **a** and **b** according to the identified cycle.	$A = 1.5$ $\lvert a \rvert = 1.5$ $a = \pm 1.5$ Based on the identified cycle, you can deduce that a = 1.5. $p = \frac{2\pi}{\lvert b \rvert}$ $10 = \frac{2\pi}{\lvert b \rvert}$ $b = \pm\frac{\pi}{5}$ Based on the identified cycle, you can deduce that $b = \frac{\pi}{5}$.
3. Determine the value of parameters **h** and **k**.	Since the coordinates of the starting point of the identified cycle are (-2, 5), the value of **h** is -2 and that of **k** is 5.
4. Write the rule of the function obtained.	$f(x) = 1.5\sin\frac{\pi}{5}(x + 2) + 5$ The rule of this function could also be written as $f(x) = 1.5\cos\frac{\pi}{5}\left(x - \frac{1}{2}\right) + 5$.

OR $-1.5\sin -\frac{\pi}{5}(x+2)+5$