Trigonometric Equations

The objective of solving a trigonometric equation is to determine the value of the variable, which in this case is the angle over a given interval

Ex: If sin x = 1/2

then x is the $\sin^{-1}(x)$

i.e it is the angle whose sin value is equal to 1/2

From the trig circle, you can see that the angle is π/6 and 5π/6

over an interval [0-2π]

What is the solution of the above equation over an interval of [0-4π]?

Ans: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{6}+2\pi, \frac{5\pi}{6}+2\pi$

simplify $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

What is the solution to the above equation over all Real Numbers

Ans:

$$\left[\frac{\pi}{6}+2\pi n\right] \cup \left[\frac{5\pi}{6}+2\pi n\right]$$

where

$n \in \mathbb{Z}$

n belongs to integers

Ex2:

$$3\sin x - 2\cos^2 x = 0 \quad \text{where } x \in [0, 2\pi]$$

When you have multiple trig ratios in an equation, always try to switch to one consistent ratio using your trig identities.

$$\because \sin^2 x + \cos^2 x = 1$$
$$\cos^2 x = 1 - \sin^2 x$$

$$3\sin x - 2(1 - \sin^2 x) = 0$$

$$3\sin x - 2 + 2\sin^2 x = 0$$

Let $\sin x = p$

$$3p - 2 + 2p^2 = 0$$
$$2p^2 + 3p - 2 = 0$$
$$2p^2 + 4p - p - 2 = 0$$
$$2p(p+2) - 1(p+2) = 0$$
$$(p+2)(2p-1) = 0$$

$p + 2 = 0$

$p = -2$

$\sin x = -2$

$\emptyset$

because $\sin x$ cannot be lower than -1

$2p - 1 = 0$

$p = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$x = \sin^{-1}(1/2)$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Ex 3:-

$$\tan^2 x + 3\sec x \tan x - \sec^2 x = 1$$

over $\mathbb{R}$

① Convert to same Ratio

$$\frac{\sin^2 x}{\cos^2 x} + 3 \cdot \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} - \frac{1}{\cos^2 x} = 1$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{3\sin x}{\cos^2 x} - \frac{1}{\cos^2 x} = 1$$

Restriction $\cos^2 x \neq 0$

$\cos x \neq 0$

$$\frac{\sin^2 x + 3\sin x - 1}{\cos^2 x} = 1$$

cross-multiply.

$$\sin^2 x + 3\sin x - 1 = \cos^2 x$$

$$\sin^2 x + 3\sin x - 1 = 1 - \sin^2 x$$ (change to the same Ratio)

$$\sin^2 x + \sin^2 x + 3\sin x - 1 - 1 = 0$$

$$2\sin^2 x + 3\sin x - 2 = 0$$

$$2p^2 + 3p - 2 = 0$$

$$2p^2 + 4p - p - 2 = 0$$

$$2p(p+2) - 1(p+2) = 0$$

$$(p+2)(2p-1) = 0$$

$p + 2 = 0$	$2p - 1 = 0$
$p = -2$	$p = \frac{1}{2}$
$\sin x = -2$	$\sin x = \frac{1}{2}$
$x = \varnothing$	$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$$x = \left[\frac{\pi}{6} + 2\pi n\right] \cup \left[\frac{5\pi}{6} + 2\pi n\right]$$

$n \in \mathbb{Z}$

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