Trigonometric Formulas

$\sin(a + b) = \sin a \cos b + \sin b \cos a$

$\cos(a + b) = \cos a \cos b - \sin a \sin b$

$\tan(a + b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$

Ex 1: Knowing that a and b are 60 and 30 degrees, verify that

$\sin(a + b) = \sin a \cos b + \sin b \cos a$

$\sin(60^\circ + 30^\circ) = \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$\sin 90^\circ = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= 1 \qquad \frac{3}{4} + \frac{1}{4} = 1$

$\therefore$ LHS = RHS $\therefore$ QED

$\sin(a - b) = \sin a \cos b - \sin b \cos a$

$\cos(a - b) = \cos a \cos b + \sin a \sin b$

$\tan(a - b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

Prove that

$\sin(\pi - t) = \sin t$

$\sin(a - b) = \sin a \cos b - \sin b \cos a$

$\sin(\pi - t) = \sin\pi \cos t - \sin t \cos\pi$

Read $\sin\pi$ and $\cos\pi$ from the trig circle.

$(0)(\cos t) - \sin t(-1)$

$\sin t$

$= RHS$.

Prove that

$\cos(\pi - t) = -\cos t$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\cos(\pi - t) = \cos\pi\cos t + \sin\pi \sin t$.

$-1 \cos t + 0 \cdot \sin t$

$= -\cos t$

P 254 - 255

Q 1, 2, 3, 4

Finding the exact value of angles made of two remarkable angles.

Recall: A remarkable angle is an angle on the trig circle.

Ex: Find the exact value of

$\sin 105^\circ$

$\sin(60+45)$

$\therefore \sin(a+b) = \sin a\cos b + \sin b\cos a$

$\therefore \sin(60+45) = \sin 60\cos 45 + \sin 45\cos 60$

Read Trig $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

Properties of Radicals.

	Ex
$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$	$\sqrt{16 \times 9} = \sqrt{16} \times \sqrt{9}$
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\frac{16}{100}} = \frac{\sqrt{16}}{\sqrt{100}}$
$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$	$\sqrt{64+36} \neq \sqrt{64} + \sqrt{36}$
$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$	$\sqrt{100-64} \neq \sqrt{100} - \sqrt{64}$

Reducing a Radical.

Ex: $\sqrt{80} = \sqrt{16 \times 5}$

$= \sqrt{16} \times \sqrt{5}$

$\boxed{4\sqrt{5}}$

Rationalizing a denominator.

Ex: $\frac{1}{\sqrt{2}}$ has an irrational number as its denominator.

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$

Find the exact value

of $\tan \frac{23\pi}{12}$

① convert $\frac{23\pi}{12}$ into degrees. $= 345^\circ$

② Find two Remarkable angles adding up to 345°

$345^\circ = 300 + 45$

$315 + 30$

$120 + 225$ .... etc

Does NOT matter which option you take

$\tan 345 = \tan(300+45)$.

$= \frac{\tan 300 + \tan 45}{1 - \tan 300 \tan 45}$

side note $\tan x = \frac{\sin x}{\cos x}$

$\tan 300^\circ = -\frac{\sqrt{3}}{2} \div \frac{1}{2} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1}$

$\boxed{\tan 300^\circ = -\sqrt{3}}$

$\tan 45^\circ = \frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2} = 1$

$= \frac{\tan 300 + \tan 45}{1 - \tan 300 \tan 45}$

$\frac{-\sqrt{3}+1}{1-(-\sqrt{3})(1)} = \boxed{\frac{-\sqrt{3}+1}{1+\sqrt{3}}}$

P 255 all of it!