Trigonometric Identities

We have established earlier that

$\cos^2 x + \sin^2 x = 1$

by using the unit circle with the Pythagoras theorem.

Claim:

$$\sec^2 x = 1 + \tan^2 x$$

Proof:

We know that $\sec x = \frac{1}{\cos x}$ (by def)

" " that $\tan x = \frac{\sin x}{\cos x}$

$$\sec^2 x = 1 + \tan^2 x$$

$\therefore$

$$\frac{1}{1} + \frac{\sin^2 x}{\cos^2 x}$$ (create common den)

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x}$$ since $\sin^2 x + \cos^2 x = 1$

$$\frac{1}{\cos^2 x} = \sec^2 x$$

$\therefore$ LHS = RHS

QED

Claim:

$$1+\cot^2 x = \csc^2 x$$

$\because \tan x = \frac{\sin x}{\cos x}$

$\therefore \cot x = \frac{\cos x}{\sin x}$

$$\frac{1}{1} + \frac{\cos^2 x}{\sin^2 x}$$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin^2 x}$$

$$\frac{1}{\sin^2 x} = \csc^2 x$$

$\therefore$ LHS = RHS

QED.

p247
Entire Page.

$$\frac{2}{4} + \frac{10}{4} = \frac{2+10}{4}$$