Logarithms

The inverse of an exponential function is called a logarithmic function. When dealing with HUGE numbers the logarithm is very helpful as it makes mathematical operations manageable

The equivalence

if $m = c^n$ <=> $n = \log m_c$

$c > 1$ $c \neq 1$

$\boxed{\log_c 1 = 0}$

Ex:

Ans $8 = 2^3$ exp. base

then $\log_2 8 = 3$ (log of 8 to the base 2 equals 3)

base

Ex2: $20 = 9^n$

$\log_9 20 = n$

Ex3: $17 = 13^{2x}$

$\Rightarrow \log_{13} 17 = 2x$

$$\log_2 32 = 5$$

base → 2

push 5 over the 2

Remove log.

$$\Rightarrow 32 = 2^5$$

$$\log_2 \frac{1}{16} = -4$$

$$\frac{1}{16} = 2^{-4}$$

Default Base: Of a log in a calculator is 10 (decimal system)

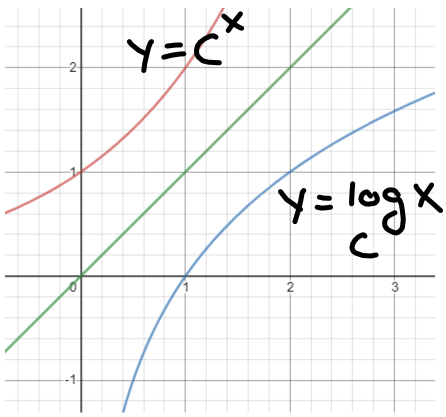
log natural has a base of e, which is an irrational number

2.71828...

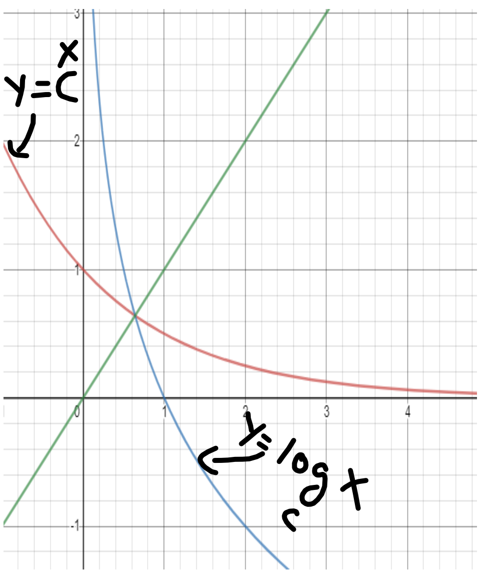
Basic Logarithmic Function

The inverse of the basic exponential function in base c, $y = c^x$ is a function called the logarithmic function written $y = \log x_c$

$c > 1$



$0 < c < 1$



Study of a Function

Regardless of the base, we have

- domain = {All positive real numbers except for 0}
- Range= All real numbers
- The function has no extrema
- The function has a vertical asymptote x=0

Change of Base Law

The default base of a logarithm (in a calculator is 10)

if it is a log natural (ln, then the base is e)

If we want to calculate log of a number to another base,

then change of base law must be used.

$$\log m_c = \frac{\log_n m}{\log_n c} \qquad n = \text{new base}.$$

$$\log_7 26 = \frac{\log_{10} 26}{\log_{10} 7} = 1.6743\ldots$$

$$\log_3 15 = 2.4649\ldots$$

Sketching a logarithmic function

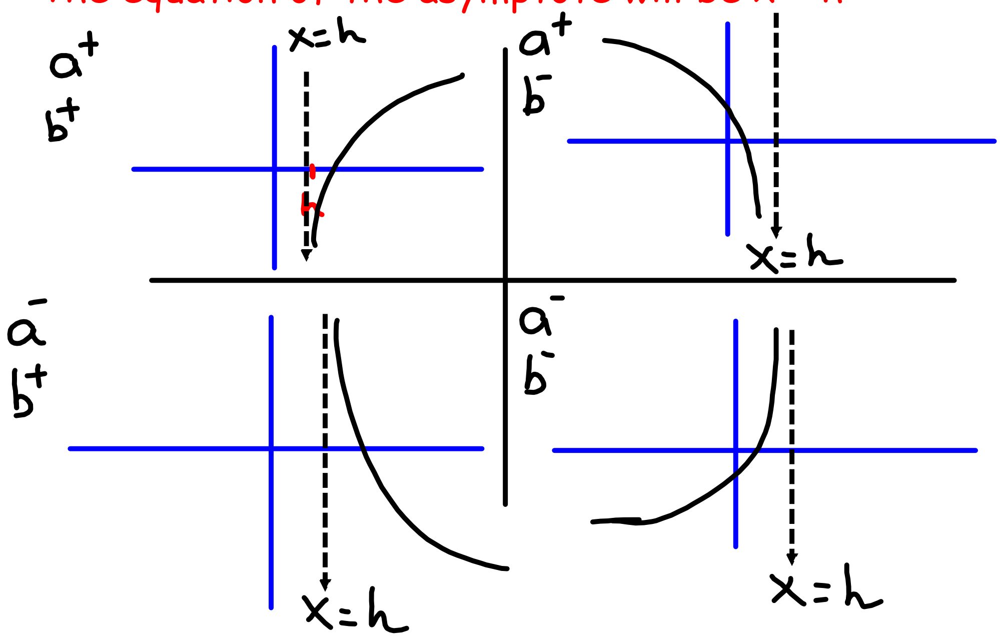
$f(x) = a\log_c(b(x-h)) + k$

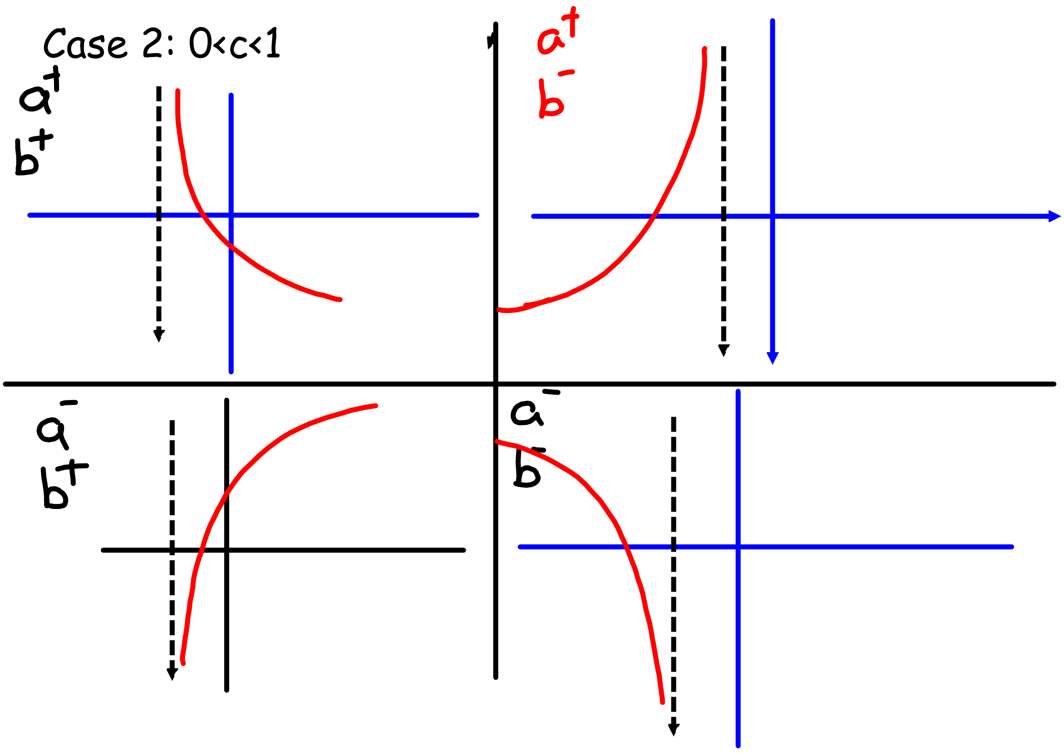
Case 1: If c>1

there are 4 possible sketches depending on the sign

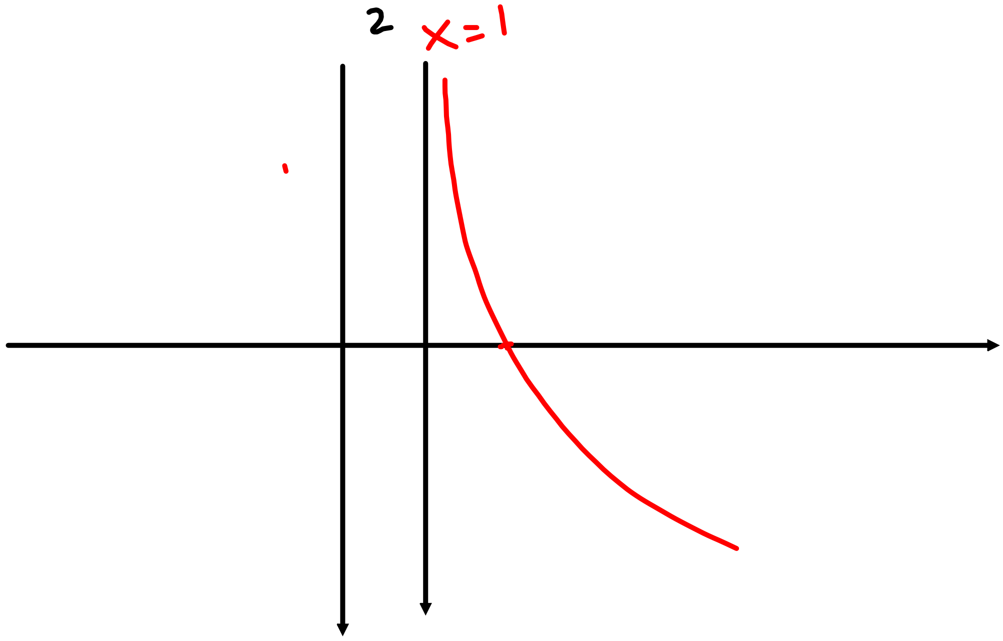
of a and b.

The equation of the asymptote will be x = h

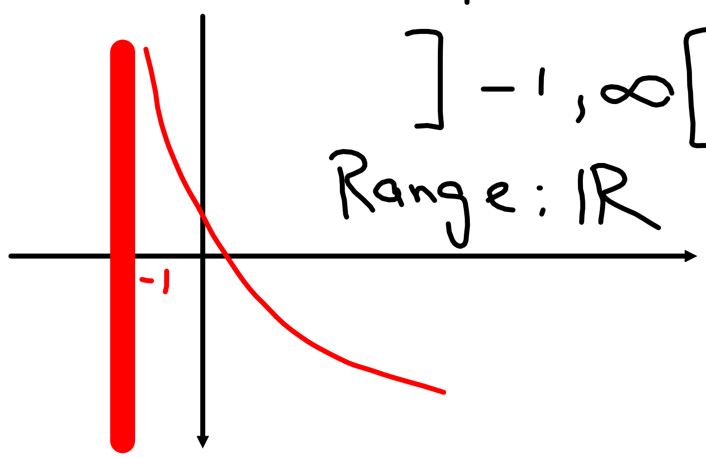



Case 2: 0<c<1
a+
b+
a+
b-
a-
b+
a-
b-

$f(x) = 2\log_{\frac{1}{2}} 2(x-1) + 4$



$f(x) = -2\log_2(x+1) + 4$



Domain

$]-1, \infty[$

Range: $\mathbb{R}$

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Q10, 11 (ignore b).

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Q12 (b)✓✗