Mathematical Expectation / Expected Value

A weighted average based on the probability of an experiment

M.E. = ($Probability_1$ x $Outcome_1$) + ($Probability_2$ x $Outcome_2$) + ($Probability_3$ x $Outcome_3$) …

If M.E. is positive → Game favors the player
If M.E. negative → Game does NOT favor the player
If M.E. = 0 → Game is fair

Example 1

A friend offers to play a game with you. You have to pay $2 to play and then roll a 6-sided die. If you roll a 6, you win $5 and win your money back. If you roll a 5, you get your money back and if you roll any other number, you lose your bet.

Should you play this game?

Outcome	Probability	NET Value (Win – Bet)	P x V
6	$\frac{1}{6}$	7-2 = 5	$\frac{5}{6}$
5	$\frac{1}{6}$	2-2 = 0	0
1,2,3,4	$\frac{4}{6}$	0 – 2 = -2	$\frac{-4}{3}$

M.E. = Sum of P x V

$$\text{M.E.} = \frac{5}{6} + 0 + \frac{-4}{3} = -0.5$$

Therefore the game does not favor the player

Example 2

A company makes hockey sticks. These sticks can be sold wholesale for a profit of \$3 each. The sticks can also be sold retail for a profit of 5\$ each. The sticks can also be defective, resulting in a loss of \$15 each.

The company estimates that 12% of their sticks are rejected, 40% are sold wholesale, and 48% are sold retail.

What is the company's **expected** profit?

Outcome	Probability	NET Value (Win – Bet)	P x V
Rejected	0.12	-15	-1.8
Wholesale	0.40	3	1.2
Retail	0.48	5	2.4

M.E. = -1.8 + 1.2 + 2.4

M.E. = 1.8

Example 3

A community organization holds a fundraiser raffle and sells 6000 tickets for $5 each. First prize is $10 000, second prize is $2000 and third prize is $1000. Is this a fair raffle?

Outcome	Probability	NET Value (Win – Bet)	P x V
1st place	$\frac{1}{6000}$	10000 – 5 = 9995	$\frac{1999}{1200}$
2nd Place	$\frac{1}{6000}$	2000 – 5 = 1995	$\frac{133}{400}$
3rd Place	$\frac{1}{6000}$	1000 – 5 = 995	$\frac{199}{1200}$
Rest of tickets	$\frac{5997}{6000}$	0 – 5 = -5	$\frac{-1999}{400}$

M.E. = $\frac{1999}{1200} + \frac{133}{400} + \frac{199}{1200} + \frac{-1999}{400}$

M.E. = $\frac{-17}{6}$ or -2.83

Game is NOT fair because M.E. does NOT equal 0

Example 3

A roulette wheel has 37 slots numbers 0 through 36. If you pick a winning number, you get your money back, plus 35 times the amount you bet. Joan places a $20 bet on a number

How much can Joan expect to win? Is it worth her while to play this game?

Outcome	Probability	NET Value (Win – Bet)	P x V
Winning Number	$\frac{1}{37}$	720 – 20 = 700	$\frac{700}{37}$
Losing Number	$\frac{36}{37}$	0 – 20 = -20	$\frac{-720}{37}$

M.E. = $\frac{700}{37} + \frac{-720}{37}$

M.E. = $\frac{-20}{37}$ or -0.54

Joan can expect to win $ -0.54 and it is not worth her time.

Example 4

A game costs $10 to play. You roll a die and the roll determines the amount of money you win.

If you roll an odd number, you lose your bet.

If you roll a 2 or a 6, you get your bet back.

If you roll a 4, you win $20 plus you get your bet back.

Is this a game fair?

Outcome	Probability	NET Value (Win – Bet)	P x V
1, 3, 5	$\frac{3}{6}$	0 – 10 = -10	-5
2, 6	$\frac{2}{6}$	10 – 10 = 0	0
4	$\frac{1}{6}$	30 – 10 = 20	$\frac{10}{3}$

M.E. = -5 + 0 + $\frac{10}{3}$

M.E. = $\frac{-5}{3}$ or -1.67

The game is not fair because the ME is not equal to 0

Example 5

You pay $5 and randomly draw a card from a standard 52-card deck.

If you draw an 7, you win four times your bet.

If you draw a face card, you win twice your bet.

If you draw any other card, you lose.

If this game fair?

Outcome	Probability	NET Value (Win – Bet)	P x V
7	$\frac{4}{52}$	20 – 5 = 15	$\frac{15}{13}$
Face	$\frac{16}{52}$	10 – 5 = 5	$\frac{20}{13}$
Other	$\frac{32}{52}$	0 – 5 = -5	$\frac{-40}{13}$

M.E. = $\frac{15}{13} + \frac{20}{13} + \frac{-40}{13}$

M.E. = $\frac{-5}{13}$ or -0.38

Game is NOT fair because M.E. does NOT equal 0

When the expected value is already known, but the bet or a prize amount is unknown, we must work backwards to find it…

Example 1

In a game of chance, you bet a certain amount to roll a die.

If you roll a 1, you win $10.

If you roll a 6, you win $5

If you roll anything else, you lose.

How much should you bet to make this game fair?

Outcome	Probability	NET Value (Win – Bet)	P x V
1	$\frac{1}{6}$	10 – x	$\frac{5}{3} - \frac{1}{6}x$
6	$\frac{1}{6}$	5 – x	$\frac{5}{6} - \frac{1}{6}x$
2, 3, 4, 5	$\frac{4}{6}$	0 – x = -x	$\frac{-4}{6}x$

$$\text{M.E.} = \frac{5}{3} - \frac{1}{6}x + \frac{5}{6} - \frac{1}{6}x + \frac{-4}{6}x$$

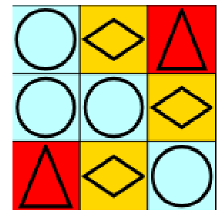
$$0 = \frac{5}{2} + -1x$$

$$1x = \frac{5}{2} \text{ or } 2.5$$

You should bet $2.50

Example 2

A game of chance involves opening one of nine doors. Behind these doors are 4 circles, 3 rhombuses and 2 triangles. Players must bet \$5 to play.



If a circle is revealed, players lose their bet.

If a rhombus is revealed, players win \$2 and keep their bet.

If a triangle is revealed, players win a certain amount of money and keep their bet.

The game is said to be fair. How much money does a player win for choosing a triangle?

Outcome	Probability	NET Value (Win – Bet)	P x V
Circle	$\frac{4}{9}$	0 – 5 = -5	$\frac{-20}{9}$
Rhombus	$\frac{3}{9}$	7 – 5 = 2	$\frac{2}{3}$
Triangle	$\frac{2}{9}$	x	$\frac{2}{9}$ x

$$\text{M.E.} = \frac{-20}{9} + \frac{2}{3} + \frac{2}{9}x$$

$$0 = \frac{-14}{9} + \frac{2}{9}x$$

$$\frac{14}{9} = \frac{2}{9}x$$

X = 7

The player will win \$7