Probability

- Probability (P) is the mathematics of chance
- Probability can be expressed as a fraction
- Probability can be expressed as a ratio (odds for : odds against)
- Probability can be expressed as a percent / decimal (chance of rain is 40%)

Sample Space / Universal Set

- List of **all** the possible outcomes
- Omega Symbol is used to represent sample space / universal set : $\Omega = \{ \dots \}$

Examples:

What is the universal set for rolling a 6 sided die?

$\Omega = \{1, 2, 3, 4, 5, 6\}$

What is the universal set for tossing a coin?

$\Omega = \{heads, tails\}$

What is the universal set for tossing 2 coins?

$\Omega = \{heads / heads, heads / tails, tails / heads, tail / tails\}$

What is the universal set for rolling two 6 sided dice?

$\Omega =\{ 1\ 1, 1\ 2, 1\ 3, 1\ 4, 1\ 5, 1\ 6, 2\ 1 , 2\ 2, 2\ 3, 2\ 4, 2\ 5, 2\ 6, 3\ 1, 3\ 2, 3\ 3, 3\ 4, 3\ 5, 3\ 6, 4\ 1, 4\ 2, 4\ 3, 4\ 4, 4\ 5, 4\ 6, 5\ 1, 5\ 2, 5\ 3, 5\ 4, 5\ 5, 5\ 6, 6\ 1, 6\ 2, 6\ 3, 6\ 4, 6\ 5, 6\ 6\}$

Multiplication Rule

When an experiment is composed of more than 1 step or done more than once, we can multiply the number of outcomes in each game to determine the number of total outcomes

Example:

You roll a 6-sided dice and then toss a coin

How many outcomes would there be?

There are 6 outcomes for the dice (1, 2, 3, 4, 5, 6) and there are 2 outcomes for the coin (Heads or Tails)

Multiplication Rule: 6 outcomes x 2 outcomes = 12 possible outcomes

Tom has 3 exams coming up. If we use P to represent a passing exam and F to represent a failing exam, how many possible outcomes would Tom have?

Exam #1 – 2 outcomes → pass or fail

Exam #2 – 2 outcomes → pass or fail

Exam #3 – 2 outcomes → pass or fail

2 x 2 x 2 = 8 possible outcomes

Ω = {PPP, PPF, PFP, PFF, FFF, FFP, FPF, FPP}

The multiplication rule is a good way to check to see if you are missing any outcomes in your sample set / universal set.

Calculating Probability

Probability is represented by numbers ranging between 0 and 1 (when using fractions or decimals) or 0 and 100% (when using percentages)

To calculate probability we use the following formula:

$$P = \frac{number\ of\ desired\ outcomes}{total\ number\ of\ outcomes}$$

Examples:

What is the probability of rolling a 5?

Ω = {1, 2, 3, 4, 5, 6}

$P = \frac{1}{6}$ or 0.17

What is the probability of rolling a number greater than 4?

Ω = {1, 2, 3, 4, 5, 6}

$P = \frac{2}{6}$ or 0.33

What is the probability of rolling a number less than 7?

Ω = {1, 2, 3, 4, 5, 6}

$P = \frac{6}{6}$ or 1

What is the probability of rolling a 12?

Ω = {1, 2, 3, 4, 5, 6}

$P = \frac{0}{6}$ or 0

You randomly pick a card from a standard 52-card deck.

What is the probability of drawing:

A black card?

P = $\frac{1}{2}$ or 0.5

Half the cards are black and the other half are red

A face card? (NOTE – I consider any card without a number on it a face card)

P = $\frac{16}{52}$ or 0.31

Jack Queen King Ace …. For Hearts, Diamonds, Spades and Clubs … 4 x 4 = 16

A red king?

P = $\frac{2}{52}$ or 0.04

King of Hearts and King of Diamonds

A joker?

P = $\frac{0}{52}$ or 0

There are no jokers in a 52 card deck

- A bag contains 9 marbles: 3 red, 1 white and 5 blue. You select a marble at random.

What is the probability of drawing a red?

Ω = {RED, RED, RED, WHITE, BLUE, BLUE, BLUE, BLUE, BLUE}

$P = \frac{3}{9}$ or 0.33

What is the probability of drawing a blue?

Ω = {RED, RED, RED, WHITE, BLUE, BLUE, BLUE, BLUE, BLUE}

$P = \frac{5}{9}$ or 0.56

Logical Connectors

In some situations, you are asked to determine the probability of multiple events. There are key words to help make the calculations easier:

The word 'and' implies multiplication of probability

The word 'or' implies addition of the probability

Examples:

- A bag contains 9 marbles: 3 red, 1 white and 5 blue. You select 2 marble at random, in succession, replacing the previous marble.

What is the probability of drawing 2 reds?

Hint → this can be reworded to drawing 1 red **and** then another red

$P = \frac{3}{9} \times \frac{3}{9} = \frac{1}{9}$ or 0.11

What is the probability of picking a blue **or** a white?

$P = \frac{5}{9} + \frac{1}{9} = \frac{2}{3}$ or 0.67

- A bag contains 6 marbles: 2 green and 4 yellow. You select 2 marble at random, in succession, without replacing the previous marble.

What is the probability of drawing a green and then a yellow?

$P = \frac{2}{6} \times \frac{4}{5} = \frac{4}{15}$ or 0.27

The 2nd denominator changed because we took out a marble on the first draw.

What is the probability of drawing 3 yellow?

Hint → this can be reworded to drawing 1 yellow, and then another yellow and then another yellow.

$P = \frac{4}{6} \times \frac{3}{5} = \frac{2}{4} = \frac{1}{5}$ or 0.20

The numerator changes as well here because we are taking away yellows and then trying to draw yellows again.

- A box contains 6 marbles: 2 blue, 1 green and 3 red. Two marbles are drawn in succession and without replacement.

What is the probability that both marbles are the same colour?

You are looking to draw RR or BB

Note → Reword this to drawing a red and then a red OR a blue and then a blue

$P = \left(\frac{2}{6} \times \frac{1}{5}\right) + \left(\frac{3}{6} \times \frac{2}{5}\right) = \frac{4}{15}$ or 0.27